# NUMERICAL EVALUATION OF INTEGRALS OF ANALYTIC FUNCTIONS OF ONE COMPLEX VARIABLE

The numerical evaluation of integrals of complex valued analytic functions is a relatively recent topic of investigation compared to the approximation of real integrals. This interest has been growing quite rapidly because of often occurrence of contour integrals in different fields of scientific investigation. The definition of the existence of integrals of analytic function given by

$$J(f) = \int_{\Gamma} f(z)\, dz \tag{1}$$

where $f$ is analytic in a disk

$$\Omega = \{z : |z - z_0| \le r\}, \tag{2}$$

and $\Gamma$ is an open contour in the disk $\Omega \subset C$ in standard texts on complex analysis (Ref. Goursat (1959), Moretti (1968), Conway (1980) etc.).

One of the properties of analytic functions suggests that if a function $f$ is analytic in a convex domain (disk is a convex domain), then the integral $J(f)$ along $\Gamma$ and along a straight line path $L$ joining the end points of $\Gamma$ are the same. So, we shall be only concerned with numerical evaluation of the complex integral of the type

$$J(f) = \int_{L} f(z)\, dz \tag{1.6.3}$$

where $L$ is a directed line segment from the point $z_0$-$h$ to $z_0$+$h$.

Scarborough (1930) gives the extension of the Simpson's 1/3$^{rd}$ rule meant for the integral $J(f)$ and it is given by

$$Q_S(f) = \frac{h}{3}\{f(z_0 - h) + 4f(z_0) + f(z_0 + h)\}. \tag{1.6.4}$$

The rule $Q_S(f)$ has degree of precision 3. However, the first paper which appeared for the approximate evaluation of the complex integral $J(f)$ of the interpolatory type using 5 nodes inside the disk $\Omega$ is due to Birkhoff and Young (1950). Later Lether (1976) using a conformal mapping $w \to z_0 + ht$ has given a transformed rule involving three nodes of degree of precision 5 for the numerical evaluation of the integral $J(f)$.

Other rules meant for the numerical approximation of the one dimensional complex contour integral $J(f)$ are due to the following: Tosic (1978), Acharya and Pattnaik (1984), Lyness and Delves (1967), Acharya and Das (1983), Acharya and Mahapatra (1992), Acharya and Nayak (1996), Acharya and Nayak (1996), Acharya, Acharya and Pati (2007), Milovanovic and Dordevic (1982) and so on.

**GENERATION OF COMPOUND RULES**

In general for generating a compound rule based on any basic rule $Q(f)$ (say) the contour $L$ which is a directed line segment from the point $A=z_0-h$ to $B=z_0+h$ we subdivide the line segment into $N$ equal parts of length $2|h|/N$ and apply the rule $Q(f)$ to each partial line segment. On summing up the partial results, the outcome is the desired compound approximation $Q^C(f)$ to the given integral $I(f)$.

For developing compound rules based on the basic rules rule $Q_{(1)S}(f)$, $Q_{(1)L}(f)$, $Q_{(1)BY}(f)$, $Q_{(1)T}(f)$, $Q_{(1)AP}(f)$, $Q_{(1)AM1}(f)$ and $Q_{(1)AM2}(f)$ the following notations are used for the numerical approximation of the complex contour integral $I(f)$ where the directed line segment $L$ joins the points with affixes A and B.

$H = (B-A)/2$,

$L_j$ : the directed line segment joining the end points $A+2H(j-1)$ to $A+2jH$ on the path $L$,

$C(j)$ : central point of the $j$th line segment $L_j$ ,

$P(j,k)$ : the four nodes for the jth line segment $L_j$ other than the central point $C(j)$

With the help of the above notations, $C(j)$ and $P(j,k)$ are given as

$$C(j) = A+H(2j-1)/N,$$

$$P(j,k) = C(j)+(i)^{k-1}\lambda H/N,$$

$$Q(j,m) = C(j)+(i)^{m-1}\mu H/N$$

$$j=1(1)N; \quad k=1,3; \quad m=2,4; \quad i=\sqrt{-1}.$$

where $\lambda$ and $\mu$ are positive real numbers less than equal to 1. The suitable values of the parameters $\lambda$ and $\mu$ are to be assigned while implementing a compound rule.