INTRODUCTION TO NUMERICAL EVALUATION OF MULTIDIMENSIONAL COMPLEX INTEGRALS

When we extend from functions of one complex variable to functions of several complex variables, the difficulties encountered are much more compared to what it is in case of real valued functions of real variables. In this connection, the remark of Poincare' (Ref. Goursat (1959)) is noteworthy. He opines "when we undertake to extend to functions of several complex variables, the general theorems which Cauchy deduced for analytic functions of one complex variable from the consideration of definite integrals taken between imaginary limits, we encounter various difficulties".

In view of the remark concerning line segment as contours, we consider straight line segments as contours for the integration of analytic function of several complex variables also and consider the following complex multidimensional integral.

$$J_n(f) = \int_{L_1} \int_{L_2} \ldots \int_{L_n} f(z^{(1)}, z^{(2)}, \ldots, z^{(n)})\, dz^{(1)} dz^{(2)} \ldots dz^{(n)} \tag{1}$$

where f is analytic in the product domain $\prod_{j=1}^{n} \Omega_j$ where

$$\Omega_j = \{ z^{(j)} : \left| z^{(j)} - z_0^{(j)} \right| \le r_j \}, \quad j = 1(1)n, \tag{2}$$

L_j is a directed line segment in the $z^{(j)}$-plane from the point $z_0^{(j)} - h_j$ to the point $z_0^{(j)} + h_j$ and $|h_j| < r_j$.

As in the case of the real multidimensional integral, $I_n(f)$, the product rules based on one dimensional rules have been formulated by Acharya and Das (1981)

and Das, Padhy and Acharya (1981) respectively for the two dimensional case where n=2.

For the numerical approximation of the complex double integral $J_2(f)$, Das (1982) has constructed the following 9-point rule of degree of precision 5 which is a transformed rule based on the 7-point degree 5 rule due to Radon (1948). Acharya and Das (1983), Milovanovic, Acharya and Pattnaik (1986), Acharya and Mahapatra (1986) and Acharya and Mahapatra (1989) have constructed some non-product type of rules for the numerical evaluation of the integral $J_n(f)$ for dimensions n=2 and more.