COMPOUND RULES

When the range of integration is of large size, i.e. b-a is relatively large, the error $E(f)$ given by

$$E(f) = I(f) - R_{n+1}(f) \quad (1)$$

is large and the computed result is no more an approximation to the $I(f)$ because analysis of the error $E(f)$ reveals that $E(f)=O((b-a)p)$ where p is a positive integer. The rule $R_{n+1}(f)$, whether Newton-Cotes type or Gauss type is referred to as the basic rule in the sense that it is applied for the entire range of integration $[a,b]$. For closer approximation of the computed result, compound rules (also known as composite rules) are constructed and applied as they are preferable to the basic rules.

The generation of compound rules is based on the following principle which we come across in standard texts in numerical analysis *viz.* Jain, Iyengar and Jain (2003), Hildebrand (1974), Conte and DeBoor (1980), Acharya and Das (2001) etc. The range of integration $[a,b]$ is subdivided into m equal parts by the set of points $x_j = a+jh$, $j=0(1)m$ where $h=(b-a)/m$. Then we have

$$I(f) = \sum_{1}^{m} \int_{x_{j-1}}^{x_j} f(x)\,dx. \quad (2)$$

Applying a fixed basic rule for the approximation of each integral under summation sign and simplifying the right hand side of the equn. (2), we obtain

$$I(f) \approx R^C(f). \quad (3)$$

The rule $R^C(f)$ is called the compound rule corresponding to the basic rule $R_{n+1}(f)$. By reducing the step size h, the error

$$E^C(f) = I(f) - R^C(f) \qquad (4)$$

can be reduced. Because, the sum giving the compound approximation to $I(f)$ is a Riemann sum if the integrand $f(x)$ is Riemann integrable over $[a,b]$.

It is pertinent to note that for the shake of simplicity of the compound rule, the generating basic rule should be simple i.e. should involve relatively less number of nodes. Davis and Rabinowitz (1984), Jain, Iyengar and Jain (2003), Conte and deBoor (1980) give the softwares for implementing the compound rules based on the basic rules *viz.* the trapezoidal rule, the Simpson's rule etc. for the numerical approximation of the integral $I(f)$.

The compound rules are also useful for approximate evaluation of integrals of functions with certain types of singularities. In this connection, the following theorem is noteworthy.

Theorem: Let $Q_n(f)$ be a n-point basic rule for the integral $I(f)$ over the interval $[\alpha,\beta]$ which is piecewise continuous and bounded in $[\alpha,\beta]$. Then the error associated with $Q_n(f)$ satisfies:

$$E(f) = c(\beta - \alpha)^{k+1} f^{(k)}(\eta), \quad \alpha < \eta < \beta \qquad (5)$$

Using this theorem, we can get $E^C(f) \to 0 \quad as \quad h \to 0$ where $E^C(f)$ is the error associated with the compound rule approximation with step size h based on the basic rule $Q_n(f)$.