

Neutrino Oscillations with Decay: A Geometrical Analysis

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Abstract: Neutrino oscillations provide a fascinating window into the quantum nature of elementary particles and admit an elegant geometrical interpretation. In the two-flavor framework, a neutrino flavor state can be visualized as a magnetic moment-like vector whose evolution is governed by a magnetic field-like vector representing the system Hamiltonian. In the absence of dissipation, this picture leads to a simple circular motion in flavor space, closely analogous to spin precession. In this work, we extend this geometrical representation by incorporating neutrino decay in vacuum. We show that decay modifies the circular trajectory into a helical path, causing the neutrino system to effectively resemble a classical damped driven oscillator. This analogy provides an intuitive understanding of how decay induces attenuation in flavor oscillations. Remarkably, when the phase factor ξ in the decay Hamiltonian is absent, the neutrino evolution becomes mathematically equivalent to the dynamics of nuclear magnetic resonance (NMR). However, the inclusion of this phase introduces CP violation, leading to a clear deviation from the standard NMR-like behavior. We further illustrate the geometrical signatures of under-damped, critically damped, and over-damped regimes through three distinct schematic representations, offering a visually transparent classification of decay effects. Finally, we present a comparative geometrical study of neutrino oscillations in vacuum, in matter, and in the presence of decay. This unified picturization provides a simplified and intuitive framework for understanding neutrino oscillation phenomena and highlights the power of geometrical methods in elucidating complex quantum dynamics.