

Understanding Planck's Quantum After 125 Years: Direct Quantization of Electromagnetic fields of Single Photons Using Operator Method

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Abstract: For the first time, the direct first quantization of the electromagnetic fields of a single photon is done in the space of the fields by defining appropriate spin-inclusive creation and annihilation operators to find the two-level energy spectrum of a single photon of any polarization.

Key words: Single Photon, First quantization, Operator method, Photon polarization, Quantum Mechanics

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INTRODUCTION

Planck's introduction of the energy quanta 125 years ago as a hypothesis to explain black body radiation spectra played a significant role in our understanding of microscopic phenomena and in the development of modern physics in all its branches such as Atomic Physics, Nuclear Physics, Particle Physics, Condensed Matter Physics, Statistical Physics and so on [1-3]. On the basis of the huge experimental evidence in support, it is taken for granted that the basic quantum of energy is for a photon of frequency ν , is:

$$\varepsilon = h\nu \quad (1)$$

where $h = 6.625 \times 10^{34}$ J.s is the well-known Planck's constant. In Photoelectric effect, a photon gets its energy divided between work function of metal and kinetic energy of the photoelectron, while in Compton effect it gets divided into the energy of a less energetic photon and the kinetic energy of the recoil electron [4,5]. These experimental facts establish the divisibility and shareability of the energy of a photon.

However, when we use a polarizer or Quarter wave plate to get a definite Polarization (linear or circular) of the photonic beam from unpolarized state, the intensity of the beam gets reduced by Malus's Law as:

$$I(\theta) = I_0 \cos^2 \theta \quad (2)$$

where the initial intensity is I_0 and θ is the angle between the beam polarization and the polarizer axis. But, if we pass a single photon through such a polarizer, its energy does not decrease as per the angular dependence of the intensity, which is proportional to the number of photons. Rather, it has probability of passing through the polarizer given by $\cos^2 \theta$ which means that it passes with certainty only if its polarization is aligned with the polarizer axis and does not pass if they are perpendicular [6]. Nevertheless, what is important is that its energy does not suffer at all, and, when it does pass, the post-passage polarization is always aligned with the polarizer axis. This illustrates the *quantumness* of the single photon, especially, of its polarization and hence, of its electric and magnetic fields, which both should be quantizable.

Recent advances in experimental physics of single photons indicate its importance in diverse areas such as:

- (a) Foundations of quantum mechanics, for probing:
 - (i) wave-particle duality using single photon self-superposition e.g. using Mach-Zender interferometer
 - (ii) the limits of simultaneous measurability of conjugate observables
 - (iii) the Heisenberg cut using decoherence models
 - (iv) quantum entanglement and non-locality in Bell tests
- (b) Quantum information and computation, for purposes of:
 - (i) SQKD (Secure Quantum Key Distribution) to ensure secure communication using single photons as the qubits
 - (ii) Operations in long-distance communication of quantum information using quantum repeaters and networks
 - (iii) Measurements involving single photon sensitivity transcending classical sensing limits
 - (iv) Testing indistinguishability and reliability of single photon sources (SPS) using HOM (Hong-Ou-Mandel) effect in quantum optics [7]
- (c) Novel experimental applications for:
 - (i) Probing the Significance of the usually neglected magnetic field of the photon [8].
 - (ii) Developing reliable SPSs and SPDs (Single Photon Detectors) for future quantum technologies
 - (iii) Photonics, quantum optics and related applications

A deeper theoretical understanding the quantum mechanics of single photons and its relation to the classical electromagnetic fields is therefore of utmost importance [9,10]. Explicit construction of single photon states following the one-photon excited quantum vacuum has also been discussed in the literature [11,12]. The earliest approach is that of Riermann-Silberstein using the electromagnetic fields themselves to define photon wave functions [13,14]. The other approach is to use the relativistic energy-momentum relation as in Dirac equation for massless particles i.e. the Optical Dirac equation [15]. This latter approach has no problems with the spatial wave function of the photon, while the former, being in terms of the field has to rely on Fourier transformation to momentum space wave functions. A recent review of some of the problems and approaches to first quantization of single photons is given by Barnett [15] and in Chichkov [16]. A better understanding of single photon quantum mechanics is required to deal with more fundamental questions concerning classical and quantal properties of light [17].

To this end, we use the essentials from Maxwell's theory and develop an operator method similar to the treatment of the quantum mechanical simple harmonic oscillator [18], to uncover, for the first time in the last 125 years of our dealing with photons, by a direct quantization of the electromagnetic fields of a single photon.

We briefly recapitulate essentials of Maxwell's theory for electromagnetic waves in free space in the next section and then give an outline of the particle nature as proposed by Planck and Einstein before developing the direct first quantization of the photonic electromagnetic fields using the operator formalism.

Maxwell's Electromagnetic wave theory of light

According to Maxwell's electromagnetic theory, the electromagnetic waves in free space move with velocity equal to speed of light $c = (\mu_0\epsilon_0)^{-1/2} = 2.998 \times 10^8$ m/s where μ_0 is the magnetic permeability and ϵ_0 is the electric permittivity of free space, meaning, where there are no charges ($\rho = 0$) or current sources ($\mathbf{J} = \mathbf{0}$). Light is treated as an electromagnetic wave with oscillating mutually perpendicular electric and magnetic fields "pushing each other" to move in a direction perpendicular to both of them. Maxwell's equations for the electric and magnetic fields \mathbf{E} and \mathbf{B} in free space are given by (SI system):

$$\nabla \cdot \mathbf{E} = 0; \quad \nabla \cdot \mathbf{B} = 0 \quad (3)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}; \quad \nabla \times \mathbf{B} = (\mu_0\epsilon_0)\partial_t \mathbf{E} \quad (4)$$

The electric and magnetic fields satisfy the wave equations in free space:

$$\nabla^2 \mathbf{E} - (\mu_0 \epsilon_0) \partial_t^2 \mathbf{E} = 0 \quad (5)$$

$$\nabla^2 \mathbf{B} - (\mu_0 \epsilon_0) \partial_t^2 \mathbf{B} = 0 \quad (6)$$

Plane wave solutions of these equations are given by:

$$\mathbf{E} = \mathbf{E}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)]; \quad \mathbf{B} = \mathbf{B}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)] \quad (7)$$

where \mathbf{k} and ω are respectively the propagation vector and angular frequency given by:

$$k = \frac{2\pi}{\lambda}; \quad \omega = 2\pi\nu; \quad \omega = k\lambda\nu = kc \quad (8)$$

and φ_0 is the initial phase, which can, of course, be set to zero, by choosing the wave to cross the origin at $t = 0$. This initial phase can also be inserted into redefined complex amplitudes of the electric and magnetic fields $\tilde{\mathbf{E}}_0 = \mathbf{E}_0 e^{i\varphi_0}$, $\tilde{\mathbf{B}}_0 = \mathbf{B}_0 e^{i\varphi_0}$. This makes fields \mathbf{E} and \mathbf{B} themselves complex, whose real parts we have to utilize to connect to experimental results.

The equality of the phases of \mathbf{E} and \mathbf{B} as given in eq. (7) is something not very apparent in Maxwell's equations themselves, but can be inferred. It has been proposed in the literature that there should indeed be a phase difference of 90° between them, the electric field term acting as potential energy and the magnetic field term as Kinetic energy in the energy expression. Of course, it is true even in Maxwell's theory that there would be a phase difference between the fields near the source and inside material media, but it dies down with distance and vanishes in the far field limit.

From Poynting's theorem, the energy density of the electromagnetic field is given by:

$$u = \frac{1}{2} \epsilon_0 \mathbf{E}^2 + \frac{1}{2} \mu_0^{-1} \mathbf{B}^2 \quad (9)$$

and the Poynting vector \mathbf{S} (energy flowing across unit area per second) is given by:

$$\mathbf{S} = \mu_0^{-1} (\mathbf{E} \times \mathbf{B}) = \epsilon_0 c^2 (\mathbf{E} \times \mathbf{B}) \quad (10)$$

They satisfy the free space continuity equation:

$$\nabla \cdot \mathbf{S} + \partial_t u = 0 \quad (11)$$

From the curl equations eq. (3), the wave vector \mathbf{k} , electric field \mathbf{E} and magnetic field \mathbf{B} are related by:

$$\mathbf{k} \times \mathbf{E}_0 = \omega \mathbf{B}_0; \quad \mathbf{k} \times \mathbf{B}_0 = -\frac{\omega}{c^2} \mathbf{E}_0; \quad \mathbf{k} \cdot \mathbf{E}_0 = \mathbf{k} \cdot \mathbf{B}_0 = \mathbf{E}_0 \cdot \mathbf{B}_0 = 0; \quad \mathbf{E}_0 \times \mathbf{B}_0 = \frac{E_0^2}{\omega} \mathbf{k};$$

$$E_0 = c B_0 \quad (12)$$

The fields are derivable from the electric scalar potential φ and the magnetic vector potential \mathbf{A} :

$$\mathbf{E} = -\nabla\varphi - \partial_t \mathbf{A} ; \quad \mathbf{B} = \nabla \times \mathbf{A} \quad (13)$$

And, they are therefore invariant under a gauge transformation of the potentials to a new set:

$$\Phi' = \Phi - \partial_t \chi ; \quad \mathbf{A}' = \mathbf{A} + \nabla \chi \quad (14)$$

This gauge invariance of the fields has been recognized and utilized as a global U(1) symmetry of the Maxwellian field in Quantum Electrodynamics (QED), on which we will have very little to say in this article.

Particle nature of electromagnetic radiation

As per the energy-momentum relation of Special theory of relativity, the particles of electromagnetic radiation travelling at the speed of light must be massless and hence their energy has to be entirely kinetic:

$$\varepsilon = pc \quad (15)$$

where, p is the momentum of the particle of radiation (photon) given, from eq. (1), by:

$$p = h\nu/c \quad (16)$$

From eq. (10), we also infer, for a monochromatic beam, a momentum density $\mathbf{\Pi}$:

$$\mathbf{\Pi} = \frac{\mathbf{S}}{c^2} = \epsilon_0 \mathbf{E} \times \mathbf{B} = \frac{nh\nu}{c} \hat{\mathbf{p}} \quad (17)$$

The energy density given by eq. (9) is proportional to the number density n of photons in the electromagnetic radiation beam:

$$u = nh\nu = npc \quad (18)$$

We must note that, in quantum theory, this kind of exact expression for free particle energy and momentum are not entirely correct because they would

violate the uncertainty principle, unless we take infinite time for measurement of energy and have infinite position uncertainty. But such values can be taken as eigenvalues of the corresponding quantum operators, resulting from the solution of appropriate quantum mechanical equations such as Schrödinger or Klein-Gordon or Dirac equation, as the case demands. The aforesaid conditions of infinite measurement time and infinite position uncertainty are automatically satisfied for many systems *e.g.* the harmonic oscillator and such other systems, which have definite energy eigenvalues.

Therefore, there must be a way of quantizing the electromagnetic fields of a photon in a similar way to get the fixed energy eigenvalue within quantum mechanics. Before that, we mention that we can use the three-dimensional Dirac delta function to express the instantaneous number density and energy density for a single photon ($N=1$) at $\mathbf{r} = \mathbf{r}_0$ as follows:

$$n(\mathbf{r}) = N\delta(\mathbf{r} - \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0); u(\mathbf{r}) = \hbar\omega\delta(\mathbf{r} - \mathbf{r}_0) = n(\mathbf{r})\hbar\omega \quad \dots (19)$$

We will not digress into QED (Quantum Electrodynamics) because our focus is quantization of the fields of the single photon, and not on the fields as infinite-DOF systems.

Operator method for direct quantization of the fields

Introducing a normalization volume V (which of course, will finally drop out, as it should) around the instantaneous location of the photon for quantization and the well-known properties of the Pauli spin matrices for introducing spin, we write the Single Photon Hamiltonian (SPH) corresponding to eq (9) above as:

$$H = \frac{V}{2}\epsilon_0\mathbf{E}^2 + \frac{V}{2}\epsilon_0c^2\mathbf{B}^2 = \hbar\omega[(\hat{\sigma} \cdot \mathcal{E})^2 + (\hat{\sigma} \cdot \mathcal{B})^2] \quad \dots (20)$$

$$\text{where, } \mathcal{E} = \sqrt{\frac{V\epsilon_0}{2\hbar\omega}} \mathbf{E} \text{ and } \mathcal{B} = \sqrt{\frac{V\epsilon_0c^2}{2\hbar\omega}} \mathbf{B} \quad \dots (21)$$

We define the fermionic ladder operators \hat{F} and \hat{F}^\dagger by:

$$\hat{F} = \hat{\sigma} \cdot \mathcal{E} + i\hat{\sigma} \cdot \mathcal{B}; \hat{F}^\dagger = \hat{\sigma} \cdot \mathcal{E} - i\hat{\sigma} \cdot \mathcal{B} \quad \dots (21)$$

These enable us to express the SPH of eq. (20) as:

$$H = (\hat{N} + \hat{\sigma}_z)\hbar\omega; \hat{N} = \hat{F}^\dagger\hat{F} \quad \dots (22)$$

We can readily verify the fermionic nature of \hat{F} and \hat{F}^\dagger :

$$\{\hat{F}, \hat{F}^\dagger\} = 0; \hat{F}^2 = \hat{F}^{\dagger 2} = 0 \quad \dots (23)$$

and, the commutation relations:

$$[\hat{F}, \hat{F}^\dagger] = 2\hat{\sigma}_z; [\hat{N}, \hat{F}] = -2\hat{\sigma}_z\hat{F}; [\hat{N}, \hat{F}^\dagger] = -2\hat{\sigma}_z\hat{F}^\dagger \quad \dots (24)$$

The nonnegativity of the eigenvalues of the number operator \hat{N} is readily established and the energy eigenvalues are:

$$E_n = (n + \langle \hat{\sigma}_z \rangle) \hbar \omega = (n \pm 1) \hbar \omega; \quad n \geq 0 \quad \dots, \quad (25)$$

Using eq. (23) and (24), we find that the vanishing of the squared ladder operators implies that n can have only two values 0 and 2 for the degenerate positive energy eigenvalue $\hbar \omega$. The ladder operators just serve to switch values of n and flip the helicities simultaneously, keeping the energy fixed! Thus, they switch the left polarized to the right polarized and vice versa. These degenerate single photon polarization (as seen from the source) states in the $\{|n, \langle \hat{\sigma}_z \rangle \rangle\}$ basis are respectively identified as:

$$|L \rangle = |2, -1 \rangle \ \& \ |R \rangle = |0, +1 \rangle \quad \dots \quad (26)$$

We note that the quantization scheme using operator method proposed here holds equally well for all three kinds of polarizations. It is easily seen that appropriate linear superpositions of the pair of degenerate states can lead to plane polarized photons. We have to make the identification for spin as:

$$\hat{S} = \hbar \hat{\sigma}; \quad S_z = \pm \hbar \quad \dots \quad (27)$$

Also, it is to be noted that we had to use eq. (12) and eq. (17) following from Maxwell's equations in deriving the (anti)commutation relations in eq. (23) and eq. (24), which means that this method is true only for photons.

DISCUSSION

The fermionic nature of \hat{F} and \hat{F}^\dagger simply tell us that even though the spin is \hbar , the allowed projections (*i.e.* helicities) are only $\pm \hbar$ and that an SU(2) representation is perfectly valid for the photon exactly like a spin 1/2 fermion, as far as the spin-polarization states are concerned. The classical relativistic reason for the absence of the zero spin-projection is the absence of rest mass for the photon eq. (15), which however does not directly determine the SU(2) representation here, though indirectly it does so through the form of the Hamiltonian eq. (20) being just the sum of a pair of squared terms in the fields.

When we superpose a pair of arbitrarily polarized states of the same photon to get a definitely polarized photon, we have to sum the spin projections as follows:

$$S_{z1} = \pm \hbar, S_{z2} = \pm \hbar; \quad S_z = \pm \hbar \quad \dots \quad (28)$$

where, in the last step, the possibilities $S_z = 0$ and $S_z = 2$ are rejected due to the fermionic nature of \hat{F} and \hat{F}^\dagger . If we wish to consider a definitely polarized single

photon state as composed of a pair of arbitrary polarized single photon states, then as per quantum theory of addition of angular momenta, when two unit integral spins are added, the resulting spin can have values 0,1 and 2 units, out of which we have to retain here only the unit spin, as the ladder operators turned out to be fermionic, allowing for only two projections. The quantum numbers n_1 and n_2 both take the values 0 and 2 such that $n = n_1 + n_2$ also takes values 0 and 2 only. Thus, the allowed possibilities in terms of $\{|n_1, n_2; \langle \hat{\sigma}_z \rangle \rangle\}$ basis are: $\{|0,0;+1\rangle; |0,2;-1\rangle, |2,0;-1\rangle\}$ out of which the last two states are completely identical after superposition leading to the same pair of final states as in eq. (26).

Extension of this kind of result (which strictly holds for only the states of a single photon) to the similar superposition of states of a pair of identical photons, is however, impossible because of the fact that the final spin of two photons can only be 0 (antisymmetric singlet) or $2\hbar$ (symmetric quintet), considering the conservation of the total spin projection, which can either be 0 or $\pm 2\hbar$. In this case, entangled photonic states (cubits) result, rather than mere superposed states. We have to remember that these states of eq. (25) and eq. (26) are states of the single photon.

CONCLUSION

We have thus successfully first-quantized the fields of a single photon and obtained its two-state nature having the positive and negative helicities, though both are degenerate with respect to energy. The introduction of the Pauli spin matrices not only provides us with a proper description of the spin of the photon with its two projections, but also allows us to have the fermionic operators, rather than the bosonic ones for the proper quantization of the system. One can contemplate further theoretical and practical applications of these ideas in diverse areas of practical interest starting from quantum logic networks to fabrication of single photon sources as well as detectors as mentioned in the introduction.

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