Orissa Journal of Physics
ISSN 0974-8202

Vol. 32, No.1
August 2025
pp. 19-25

Revisiting the Sun's Azimuth for Pathani Samanta's *Vusamantara Surya Ghadi* and Construction of Generic *Golardha Yantra*

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Abstract: The advance treatment of *Vusamantara suryaghadi* requires declination lines which involve sun's azimuth. In this article, the well-known formula of sun's azimuth is rederived using formulas of inclination angle and azimuth which involves sun's declination, hour angle and sun's inclination. We have derived sun's azimuth as a function of sun's declination, hour angle and latitude only. Moreover, the *Golardha yantra* of Pathani Samanta is modified to make it applicable throughout the year. For this, the underlying similarity between *Golardha yantra* and Analemmatic sundials explored and the gnomon of *golardha yantra* is made movable in the north-south direction.

Keywords: Horizontal Sundial, Sun's azimuth, *Golardha yantra*, Gnomon

INTRODUCTION

Sundials are the oldest known instruments for measurement of time [1]. When the sun moves across the sky, its shadow falls on the hour markings, from which the time is measured [1]. In this article we will discuss, the advanced study on sundial as well as new modification of it. Below are given some well-known sundials:

Horizontal Sun-dial

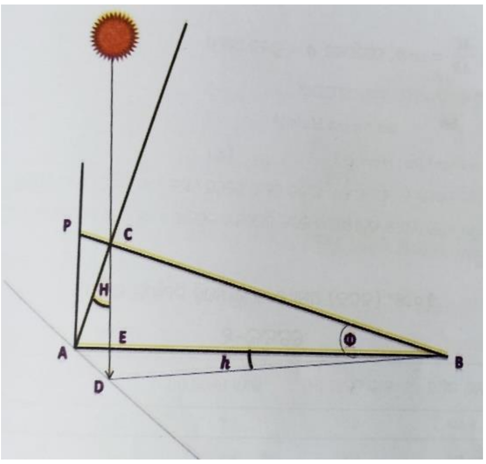

Figure 1: Ray diagram of Horizontal Sundial

Horizontal sundial consists of a horizontal plane above which a triangular gnomon is mounted. Here the gnomon is tilted at an angle equals to the latitude of that place. When the shadow of the triangular gnomon falls on the horizontal plane, from the hour lines on which it falls, the local time is noted [1].

The circular equatorial sundial is projected onto a horizontal plane in a horizontal sundial. An ellipse is formed from the circle. The tilt of the gnomon depends on the latitude of that place. The hour angles on an equatorial sundial are different from the shadow angle on the horizontal plane. We record the local time when the shadow of the triangular gnomon lines up with the horizontal plane's hour lines. The formula determines the hour angles [1, 2]:

$$\tan h = \tan H \cdot \sin \phi$$

Golardha yantra

A sphere's lower hemisphere is used to make this. A gnomon with a length equal to the sphere's radius is positioned in the centre of the hemisphere. A time-displaying strip that extends from west to east is fixed and goes through the middle (or touches the gnomon's base). The sundial should be oriented with the strip facing west-east so that it is under the sky. The point of contact of the gnomon's shadow on the strip will be used to calculate the time when it falls on the hemisphere by drawing a line perpendicular from the shadow's tip [3,4].

Analemmatic Sundial

With hour markers arranged in an elliptical pattern, this kind of sundial features a vertical gnomon. The location's latitude determines the kind of ellipse. In this case, the gnomon is not stationary and varies daily based on the sun's declination. Hour lines are therefore not present in this kind of sundial.

If M is the length of the semi-major axis of the ellipse, then the coordinates of the hour points are given by [5]

$$x = M\sin H\,, y = M\cos H\sin\phi$$

The distance the gnomon moves relative to equinox position in the north south direction is given by the formula [5]

$$Z = M\tan\delta\cos\phi$$

Theory:

Derivation of the formula:

We know [6]

$$\sin E = \sin\delta\sin\phi + \cos\delta\cos\phi\cos H$$

E =inclination angle

δ=Sun's declination

ϕ =Latitude

H =Hour angle

And if Z is the sun's azimuth, then

$$\sin Z = -\frac{\cos\delta\cdot\sin H}{\cos E}$$

$$\cos Z = \sqrt{1-(\sin Z)^2} = \frac{\sqrt{(\cos E)^2-(\cos\delta)^2(\sin H)^2}}{\cos E}$$

The numerator is

$$(\cos E)^2-(\cos\delta)^2(\sin H)^2 = 1-(\sin E)^2-(\cos\delta)^2(\sin H)^2$$
$$= 1-(\sin\delta\sin\phi+\cos\delta\cos\phi\cos H)^2-(\cos\delta)^2(\sin H)^2$$

$$= 1 - (\sin\delta)^2(\sin\phi)^2 - (\cos\delta)^2(\cos\phi)^2(\cos H)^2$$
$$- 2\sin\delta\sin\phi\cos\delta\cos\phi\cos H - (\cos\delta)^2(\sin H)^2$$
$$= 1 - (\sin\delta)^2(\sin\phi)^2$$
$$- (\cos\delta)^2\{1 - (\sin\phi)^2\}\{1 - (\sin H)^2\}$$
$$- (\cos\delta)^2(\sin H)^2 - 2\sin\delta\sin\phi\cos\delta\cos\phi\cos H$$
$$= (\sin\delta\cos\phi - \cos\delta\sin\phi\cos H)^2$$

Then $\cot Z = \frac{\cos Z}{\sin Z} = \frac{\sin\delta\cos\phi - \cos\delta\sin\phi\cos H}{-\cos\delta\cdot\sin H} = \frac{\sin\phi.\cos H - \tan\delta\cdot\cos\phi}{\sin H}$

Construction of a Generic *Golardha Yantra*:

The *Golardha yantra* developed by Pathani Samanta, is useful only during equinox, i. e. only when declination of sun is zero degree. There is a close connection between *Golardha yantra* and Analemmatic sundial. The projection of the dial plate of a *Golardha yantra* on a horizontal plane is an ellipse, which mimics the locus of hourmarks of an analemmatic sundial. In other words, the hemisphere of *Golardha yantra* along with the vertical gnomon at the centre behaves as Analemmatic sundial. But this is valid only during equinoxes. To make it generic, i.e. to make it useful throughout the year, we have to notice that Analemmatic sundial is useful throughout the year. Here, the locus of hourmarks of Analemmatic sundial remains same. There is only change in position of the gnomon of the sundial in north and south direction, from equinox position. So, to make the Golardha yantra useful for all declination of the sun, there is no inconvenience from the dial plate, but only the gnomon position is incorrect. So, a strip of material of the hemisphere is removed in the north-south direction so that the vertical gnomon will be movable in this direction. The amount of change of position from the centre is given by the formula:

$$Z = M\tan\delta\cos\phi$$

Where, the symbols have their usual meaning.

RESULTS AND DISCUSSION

(i) Figure 2 shows the graphical analysis of the functioning of the sundial. Here our analysis is limited to calculating the shadow angle of the horizontal sundial. For a particular time (i.e. for a particular hour angle), this shadow angle increases with increase in latitude of the place.

(ii) But in the more advanced stage of analysis of the horizontal sundial (or of any kind of sundial) we are not only interested in calculating the shadow angle, but we are also engaged in determining, where the tip of the shadow falls. This is called the declination line. To determine the tip coordinate in two-dimensional system, the formula of sun's azimuth is very important [7]. This we have derived in this article using our own approach.

(iii) There is an important result we got in the analysis of *Golardha yantra* and equatorial sundial, in our article. If we place our *Golardha yantra* at any latitude on earth, during equinox, it will show the correct time. And if we carefully observe the shadow of the tip of the gnomon, it passes through a half-circle, which meets with the time strip at the two ends. The *yantra* can be divided into various such strips which are inclined to the time strip at an angle ranging from zero to ninety degree.

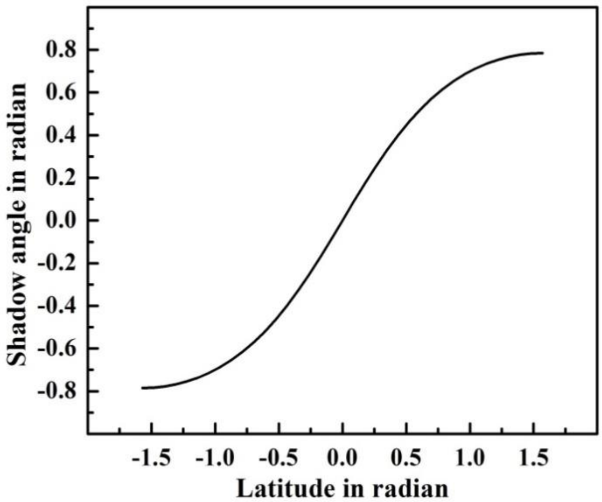

Figure 2: Variation of Shadow angle with latitude for a horizontal sundial at a constant hour angle.

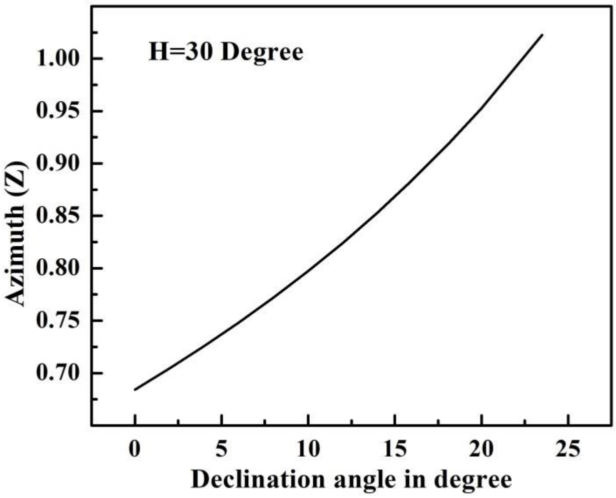

Figure 3: Variation of Sun's azimuth with declination of sun. Here, latitude of the place is taken constant at 45 degree and hour angle is taken constant at 45 degree.

(iv) If we put the *yantra* at a place with a certain latitude, it is equivalent to say that we are placing an equatorial sundial with the dial plate coinciding the locus of hourmarks of the *Golardha yantra*. If we take the horizontal projection of the locus of these hour marks (which is of course a circle), we will get what we call the ellipse of an Analemmatic sundial. So, by making way for the north south movement of the gnomon in the *Golardha yantra*, we can make it function like an Analemmatic sundial with the only exception that, the hour marks are in a three dimensional curved space.

CONCLUSION

Without sun's declination, only the time is shown by the *Vusamantara Surya ghadi*. More information about perpendicular can be obtained when sun's declination is taken into account, and Sun's azimuth is calculated. This will determine where the tip of the shadow will fall, from which, the day and the month can be determined. Similarly, the *Golardha yantra* can be used for all the days of a year, by taking into account the sun's declination. This can be done with a slight modification of the yantra which makes the stationary gnomon movable by the amount of distance similar to an Analemmatic sundial.

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