Orissa Journal of Physics
ISSN 0974-8202

Vol. 31, No. 1
February 2024
pp. 69 - 78

Gamma Function as an Optical Technique to Calculate Directionality Constant in a Glaucomatous Eye: Stiles Crawford Effect Revisited

D. K. PATTNAIK[1*], B. PANI[2], N. K SHARMA [3] A. P. SAHU [4]

[1,2,3]Siksha 'O' Anusandhan University, Bhubaneswar, OR, India, 751030
[4]Centurion University of Technology and Management, Bhubaneswar, OR, India
**pattanaikdeepak85@gmail.com*

Received 11.11.23, Accepted 3.01.24

Abstract- In an optical system without aberrations, light entering the human pupil at its centre was several times more effective in creating the perception of vision than light entering the pupil's periphery. The Stiles-Crawford effect refers to this. The optic nerve, which carries visual information from the eye to the brain, can even be damaged in the disease known as glaucoma, in which the internal eye pressure (also known as intraocular pressure, or IOP) goes up to a lethal level. The visual field percentage is used to construct an equation for a glaucomatous eye that differs from the equation for a healthy eye. In this case, the integral equation is constructed using an optical method called the gamma function. The directionality constant value of the Stiles-Crawford effect, which is acquired by performing interference on the retina, satisfies this equation. The fact that an average 0.01 contrast threshold elevation is associated with roughly 2% visual field loss for an eye with glaucoma is proven by evaluating this equation term by term using Simpson's 1/3rd rule.

Index Terms-Glaucoma, Directionality constant, Visual field loss, Integral Equation

I. Introduction

In comparison to when it is incident along the axis of the pupil, an obliquely incident light beam produces less intensity. This phenomenon is referred to as the first-kind Stiles-Crawford effect [1]. However, recent studies have shown that the SCE I memory from the retina can be eliminated by introducing a second coherent beam that is symmetrically positioned in the same meridian, close to the opposing edge of the pupil [2, 3]. An interference pattern develops in this situation. The contrast now determines how visible something is. Intraocular pressure, or IOP, can become fatally high in glaucoma, which can even cause damage to the optic nerve, which carries visual information from the eye to the brain [4]. Visual field testing [5] can be used to check for blind patches that could result from optic nerve damage brought on by glaucoma. To track the development of vision loss, this visual field examination should be repeated [6]. An equation is created while taking the visibility percentage into consideration. According to the equation's value, a glaucomatous eye's average 0.01 contrast threshold elevation is connected to a loss of roughly 2% of visual field [7]. The gamma function is defined by [8]

$$\Gamma(\lambda+1)=\int_0^\infty e^{-t}t^{\lambda}dt$$

So, $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$ [8].

$$\int_{-\infty}^{\infty} e^{-\rho r^2}dr=\sqrt{\frac{\pi}{\rho}}$$

II. Theory

In the absence of interference patterns, the traditional SCE I visibility (η) is given as [9,10]

$$\eta(r)=e^{-0.115r^2}$$

In the presence of interference, the modified visibility is given by [7, 11]

$$\eta(r)=e^{-0.115\left[-\left\{\frac{(m-1)-\sqrt{1-m^2}}{(m+1)+\sqrt{1-m^2}}\right\}r\right]^2}=e^{-0.115\left(\frac{1-m}{1+m}\right)r^2}$$

And taking contrast $m=0.5$, and distance from the centre of the pupil $r=4$ mm,we get the visibility as

$$e^{-0.115\times\frac{1}{3}\times 16}=0.544$$

So, in an eye of 4 mm pupil diameter, the visible visual field is 54.4%

As the visual field of the eye is πr^2,where $=4$ mm, 54.4% of this is given by

$$54.4\times\pi r^2=\pi\times\left(\sqrt{.54}\times r\right)^2$$

The radius corresponding to this visual field is given by

$$\sqrt{.54}r = .737 \times 4 = 2.948 \text{ mm}$$

Using the well-known gamma function, we have $\int_{-\infty}^{\infty} e^{-\rho r^2} dr = \sqrt{\frac{\pi}{\rho}}$

As the value of light intensity is zero beyond the radius of the pupil, for a healthy eye, the above equation reduces to $\sqrt{\frac{\rho}{\pi}} \int_{-4}^{4} e^{-\rho r^2} dr = 1$`

For a glaucomatous eye, the value of the integral corresponding to the gamma function is given by

$$\sqrt{\frac{\rho}{\pi}} \int_{-2.948}^{2.948} e^{-\rho r^2} dr$$

From the centre up to the above radius (up to 2.948 mm), the visual field occurs, so the integral corresponding to uncoupled energy is given by

$$\sqrt{\frac{\rho}{\pi}} \int_{-4}^{4} e^{-\rho r^2} dr - \sqrt{\frac{\rho}{\pi}} \int_{-2.948}^{2.948} e^{-\rho r^2} dr$$

The above value is also equal to the term representing the power uncoupled, which is [12],

$$1 - \sqrt{\frac{\rho_{Gl}}{\pi}} \int_{-4}^{4} e^{-\rho_{Gl} r^2} dr$$

As both are equal, we write them as,

$$\sqrt{\frac{\rho}{\pi}} \int_{-4}^{4} e^{-\rho r^2} dr - \sqrt{\frac{\rho}{\pi}} \int_{-2.948}^{2.948} e^{-\rho r^2} dr = 1 - \sqrt{\frac{\rho_{Gl}}{\pi}} \int_{-4}^{4} e^{-\rho_{Gl} r^2} dr$$

Here, $\rho_{Gl} = 0.115 \left(\frac{1-m}{1+m}\right) = \frac{0.0383}{mm^2}$, [10] we get the above as

$$\sqrt{\frac{0.0383}{\pi}} \int_{-4}^{4} e^{-0.0383 r^2} dr + \sqrt{\frac{0.115}{\pi}} \int_{-4}^{4} e^{-0.115 r^2} dr - \sqrt{\frac{0.115}{\pi}} \int_{-2.948}^{2.948} e^{-0.115 r^2} dr = 1 \quad (1)$$

III. RESULTS AND DISCUSSION

1. We will first evaluate equation (1). In this equation, the first integral on L.H.S. is

$$\sqrt{\frac{0.0383}{\pi}}\int_{-4}^{4} e^{-0.0383r^2}dr$$

Applying Simpson's $\frac{1}{3}rd$rule [12], we get $\sqrt{\frac{0.0383}{\pi}}\left(\frac{8}{3}\right)\left[2+e^{-16\times 0.0383}\right]$

$$=\sqrt{\frac{0.0383}{\pi}}\left(\frac{8}{3}\right)\left[2+e^{-0.6128}\right]=0.748127$$

Similarly, after applying Simpson's $\frac{1}{3}rd$ rule to the middle integral in L.H.S. of equation (1), it becomes [13]

$$\sqrt{\frac{0.115}{\pi}}\left(\frac{8}{3}\right)\left[2+e^{-16\times 0.115}\right]=\sqrt{\frac{0.115}{\pi}}\left(\frac{8}{3}\right)\left[2+0.15881\right]=1.101$$

And at last, applying Simpson's $\frac{1}{3}rd$rule to the last integral in L.H.S. of equation (1), we get [13]

$$\sqrt{\frac{0.115}{\pi}}\int_{-2.948}^{2.948} e^{-0.115r^2}dr=0.1913\times\frac{2(2.948)}{3}\left[2+e^{-(2.948)^2\times 0.115}\right]$$
$$=0.888$$

$$\text{So, L.H.S}=0.7481+1.101-0.888=0.9611\cong\text{R.H.S}$$

We now want to raise the contrast threshold and observe how it impacts visibility.

Because it contains the directionality constant of the glaucomatous eye, which will vary if the contrast threshold is changed, only the first integral on the L.H.S. will change. The last integral on L.H.S. will vary similarly if we alter visibility. Therefore, the middle integral is unchanged in this test. As a result, we won't consider the middle integral. The distinction between the first term and the last term for an eye in a healthy condition is

$$0.7481 - 0.888 = -0.1399$$

Now for a glaucomatous eye, if the contrast is elevated by 0.01, then the new value of [10]

$$\rho_{Gl} = \frac{1 - 0.51}{1 + 0.51} \times 0.115 = \frac{0.0373}{\mathrm{mm}^2}$$

And the first integral on L.H.S. becomes [13]

$$\sqrt{\frac{0.0373}{\pi}}\left(\frac{8}{3}\right)[2 + e^{-16\times 0.0373}] = 0.7384$$

Now, for two scenarios—case 1 for a 1% visual field loss and case 2 for a 2% visual field loss—we will determine the change in value of the last integral.

Case1

Now the visual field is 53.4% for a 1% increase in visual field loss, and the radius equivalent to this is

$$\sqrt{.534}r = .7307 \times 4 = 2.923\mathrm{mm}$$

Then the last integral is given by [13]

$$\sqrt{\frac{0.115}{\pi}} \times \frac{2(2.923)}{3}\left[2 + e^{-0.115\times(2.923)^2}\right] = 0.8840$$

Now the difference between the first integral and the last integral is

$$0.7384 - 0.8840 = -0.1456$$

Case 2

The visual field is 52.4% if the visual field loss increases by 2%, and its corresponding radius is

$$\sqrt{.524}r = .7238 \times 4 = 2.8952\mathrm{mm}$$

Then, in this case, the last integral is given by [12]

$$\sqrt{\frac{0.115}{\pi}} \times \frac{2(2.8952)}{3}\left[2 + e^{-0.115\times(2.8952)^2}\right] = 0.8783$$

Now the difference between the first integral and the last integral is

$$0.7384 - 0.8783 = -0.1399$$

So, if in equation (1), the contrast corresponding to first integral on L.H.S. is increased by 0.01 and the visual field loss corresponding to last integral of same L.H.S. is increased by 2%, the value on R.H.S. remains the same, i.e.,

$$\sqrt{\frac{0.0373}{\pi}}\int_{-4}^{4} e^{-0.0373r^2}dr + \sqrt{\frac{0.115}{\pi}}\int_{-4}^{4} e^{-0.115r^2}dr - \sqrt{\frac{0.115}{\pi}}\int_{-2.8952}^{2.8952} e^{-0.115r^2}dr = 1 \qquad (4)$$

The following graphs are drawn to show the relationships between different parameters, such as directionality constant, contrast threshold, visual field loss, etc., and to help explain why they are related:

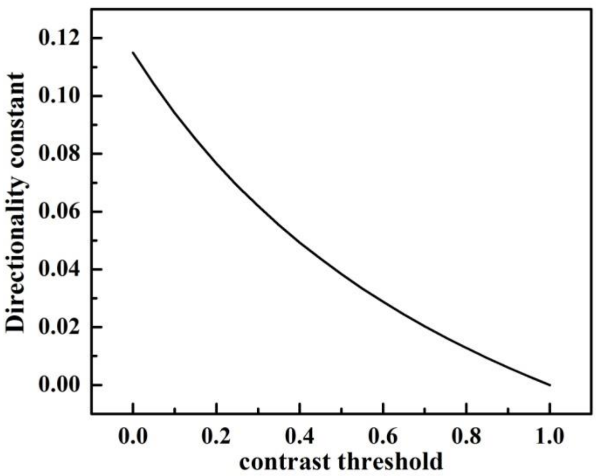

Fig. 1: Variation of directionality constant (□□□) with different contrast thresholds (□).

2. The directionality constant is represented along the y-axis of Graph 1, and the contrast threshold is plotted along the x-axis. The directionality constant is seen to decrease as the contrast threshold rises. This is due to the contrast threshold being the lowest contrast that the eye is capable of

detecting. Below this level, the eye cannot detect contrast. As a result, an eye's sensitivity begins to decline as the contrast threshold for that eye continues to rise. As a gauge of the health of the eye, directionality begins to decline as well.

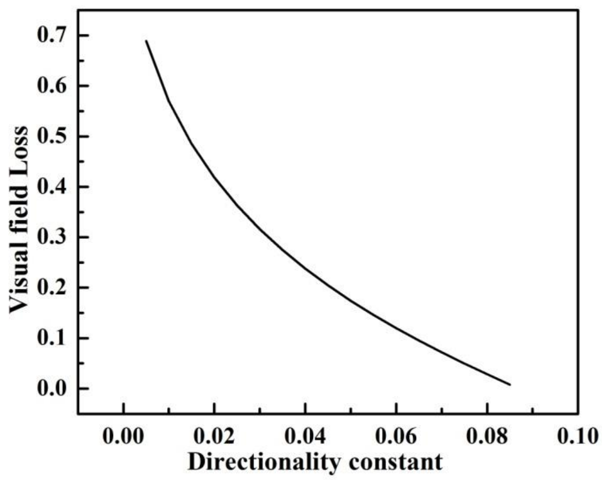

Fig.2: Relationship between visual field loss in the Glaucomatous eye and the corresponding directionality constant.

3. The directionality constant is plotted along the x-axis, and the visual field loss is displayed along the y-axis to create graph 2. The visual field loss begins to decrease as the directionality constant rises because the directionality constant of a diseased eye is lower than that of a healthy eye, and the diseased eye has a greater loss of visual field. It demonstrates the exponential relationship between the dependent (visual field) and independent variables (directionality constant) by showing that the slope of the graph declines as the value of the directionality constant increases.

IV. Conclusion

We have arrived at some significant conclusions on the loss of the visual field and directionality of a human eye using the gamma function as an optical tool in a glaucomatous eye.

1. The validity of equation (1) is established using Simpson's 1/3rd rule because the L.H.S. of equation (1) equals the R.H.S. It has been established that a glaucomatous eye can be fitted by an integral equation that holds true for two values of the directionality constant, i.e.,$\frac{0.115}{\mathrm{mm}^2}$ for healthy eye [14] and $\frac{0.0383}{\mathrm{mm}^2}$ for a glaucomatous eye.
2. Increasing visual field loss by 1% causes the limit of integration in the last term of equation (2) to change by 0.01, and increasing contrast by 0.01 causes the directionality constant in the first part of equation (2) to change from 0.0383 to 0.0373. Equation (2)'s result remains unchanged whether the first and last integrals in L.H.S. are changed; hence, it can be deduced that a 0.01 increase in contrast sensitivity corresponds to a loss of roughly 2% of visual field.
3. Once more, an optical method is constructed using integral equations, from which it is possible to derive the value of the directionality constant for various levels of visual field loss.

Acknowledgements: This article does not contain any studies with human participants or animals performed by any of the authors. All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers’ bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript. No funding was received for this research.

References

[1] N K Sharma, K Mishra, S K. Kamilla and Jatadhari K. Sharma, “Using Pupil Apodization as an Optical Technique to Obtain Retina Light

Distributions in Presence of Stiles-Crawford Effect of the First Kind" *IJMOT,*9(3), May 2014

[2] S. Castillo and B. Vohnsen," Exploring the SCE-I with coherent light and dual Maxwellian Sources," *Appl. Opt.*52(1), A1-A8(**2013**)

[3] N K Sharma, "Contrast-controlled retinal response".*Proc. SPIE* 9654, International Conference on Optics and Photonics 2015, 965428 (15 June 2015)

[4] http://www.allaboutvision.com/conditions/glaucoma.htm

[5] Ken Asakawa, Nobuyuki Shoji, HitoshiIshikawa, and Kimiya Shimizu, "New approach for the glaucoma detection with pupil perimetry" Clin *Ophthalmol,* 4:617(**2010**).

[6] Chris A. Johnson and Stephen J. Samuels, "Screening for glaucomatous visual field loss using the frequency doubling contrast test." *IOVS,* 382,413(**1997**)

[7] Nachieketa K Sharma, Kampal Mishra, Susanta K Kamilla,"Contrast-controlled retinal response as a Biomarker for a Glaucomatous Eye" *Adv.Sci. Lett.*22 (2):601-604

[8] Prahallad Chandra Naik, 'Essentials of Mathematical Physics', Kalyani Publishers,4th revised edition, 2017

[9] R.A. Applegate and V. Laxminarayanan,"Parametric representation of Stiles-Crawford functions:normal variation of peak location and directionality" *J. Opt.Soc.Am. A*10, 1611(1993)

[10] Stiles, W.S.; Crawford, B.H. "The luminous efficiency of rays entering the pupil at different points." *Proc. R. Soc. London, Ser. B* **1933,** *112,* 428-450

[11] Nachieketa K Sharma, Kampal Mishra, Sushanta K. Kamilla and Jatadhari K. Sharma,"Using Bacteriorhodopsin in Thin Film in Regulating Retinal Stimulation of a Human Eye" *Adv. Sci. Lett.* 20 $^{3}/_{4}$ 705 (2014)

[12] Nachieketa K. Sharma and Vasudevan Lakshminarayanan,”The Stiles-Crawford Effect:spot-size ratio departure in retinitis pigmentosa” *J.Mod.Opt.*63, 669-676

[13] S.S Sastry,., ‘Introductory Methods of Numerical Analysis, **2012** Prentice Hall of India

[14] Westheimer G., Directional sensitivity of the retina: 75 years of Stiles Crawford effect, *Proc.R.Soc.B.*(2008) 275, 2777-2786