

The Beauty of Malus' Law

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Abstract: This experiment aims to provide a comprehensive analysis of polarisation phenomena using Malus law, including linear, circular, and elliptical polarisation, using crossed polarisers. Malus' law provides a mathematical relationship between the intensity of polarised light and the angle between the polarising axis and the incident polarisation direction. The experiment addresses the experimental methodology for validating the Malus' law. The data collected in the experiments are compared with the theoretical predictions derived from Malus' law, and any deviations or discrepancies are analysed and discussed.

INTRODUCTION

HISTORY OF POLARIZATION

Polarization began with the discovery of polarization of light by reflection. Etienne Louis Malus (1775-1812) a French Engineer discovered the theory of intensity of polarised light. One evening in 1808 he observed the reflection of direct sunlight from windowpane through Iceland Spar and found two images of the sun. The two refracted lights varied in relative intensities as the crystal was rotated about the line of sight. That led him to formulate a law for the intensity for polarized light.

Part 1

LINEARLY POLARISED LIGHT

When unpolarized light of I_0 pass through a polaroid, then half the intensity ($I_0/2$) gets absorbed by the polaroid and another half ($I_0/2$) gets transmitted through the transmission axis of the polariser, and the emergent light is linearly polarised. This is one method of producing Plane polarised light.

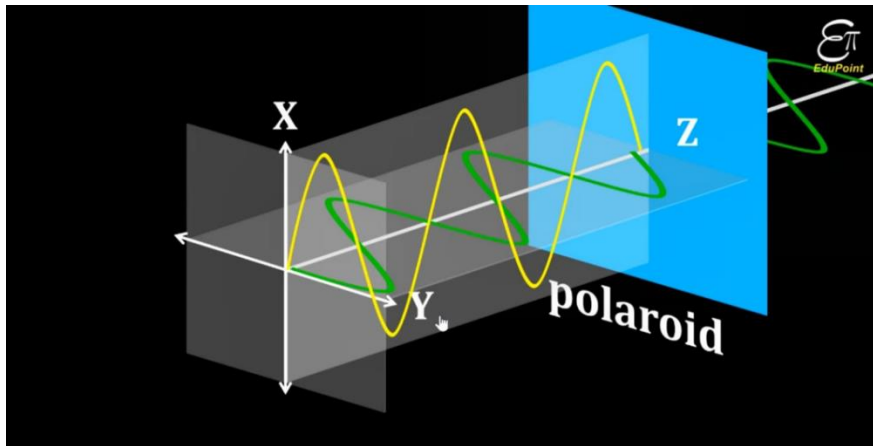


Fig1: The transmission axis of the polaroid is along the Y axis. Plane Polarised light produced by a *single* polaroid.

MALUS' LAW

Malus' law states that, “the intensity of the plane polarised light that passes through an analyser varies as the square of the cosine of the angle between the plane of the polarised light and the transmission axis of the analyser.”

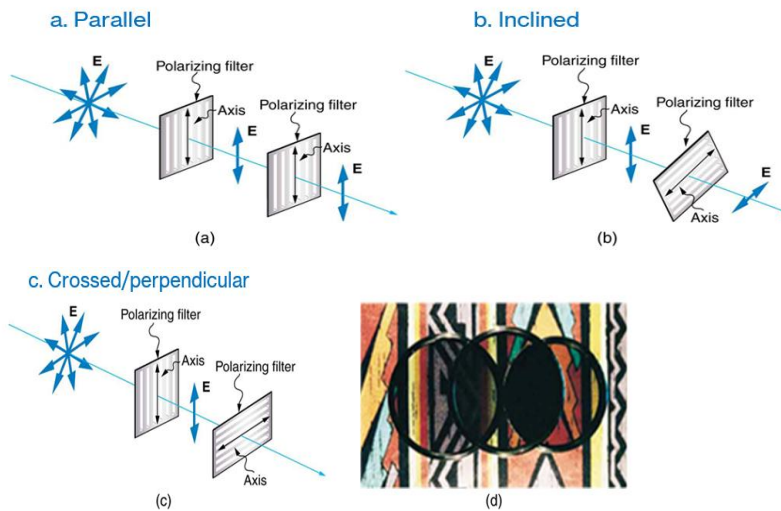


Fig 2: Intensity variation through (a) parallel polaroids – maximum intensity (b) inclined polaroids- decreased intensity and (c) crossed polaroids – no intensity (d) picture in the inset- intensity in the left overlapping region is partial while in the right overlapping region is ‘dark’!

INTENSITY MEASUREMENT

Intensity of light is measured using circuit having an LDR and a microammeter in series with a DC supply. Here, the resistance of LDR decreases with increase in intensity of light, then the current in the circuit increases. To obtain the corrected current, the background current (due to extraneous light) is subtracted from the measured current.

EXPERIMENTs

1.TWO POLAROIDs (P₁,P₂)



Fig 3: Laser, polariser, analyser and the LDR are placed coaxial at the same height. The LDR is connected to a DC supply and a milliammeter.



Fig 4: A QWP is inserted between the crossed polarizer and the analyser.

Formula 1:

I_0 be the initial intensity of the unpolarized light. I_i is the intensity after the i^{th} polaroid.

$$I_1 = I_0/2 \quad \text{after polaroid 1 (polarizer)}$$

$$I_2 = (I_0/2) \cos^2\Theta \quad \text{after polaroid 2 (analyser)}$$

Θ is the angle between the two polaroids.

For $\Theta = 0^\circ$, $I_2 = I_0/2$ (maximum, the polariser and analyser are parallel).

$\Theta = 90^\circ$, $I_2 = 0$ (minimum, the polariser and analyser are crossed).

TABLE 1: Polariser (P₁) is fixed, Analyser (P₂) is rotated.

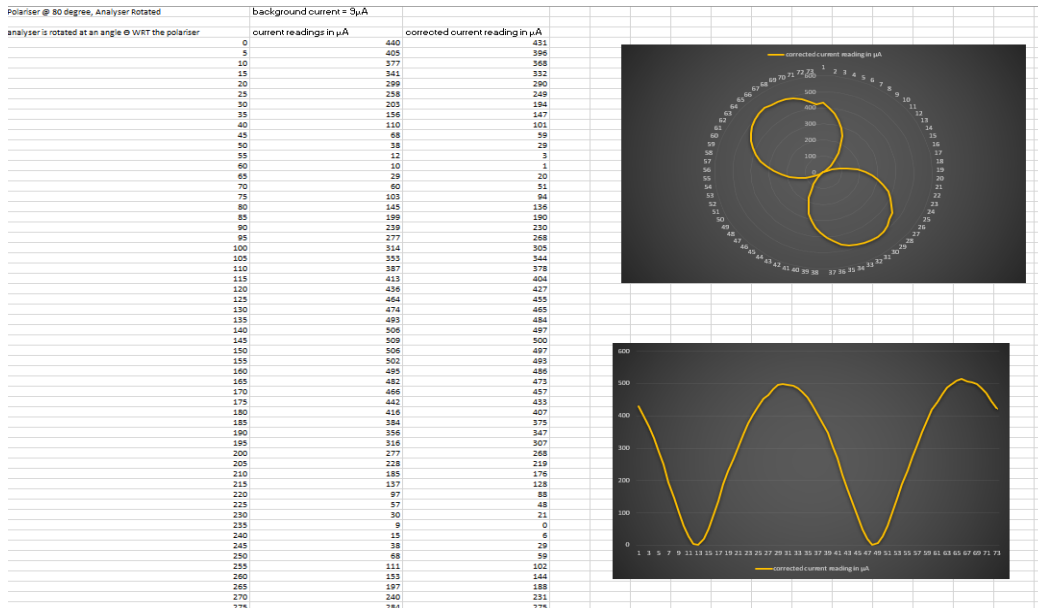


Fig 5: (bottom- plot) current versus angle shows two maxima and two minima(~zero) per cycle of rotation. (top-plot) polar graph exhibits an 'eight' representing a plane polarized light.

Result 1: A plane polarised light is detected with a rotating analyser. The intensity of light through the analyser becomes *zero* when the transmission axis of analyser is crossed twice with respect to the transmission axis of the polariser (plane of polarization of the light) and maximum twice when the transmission axis of analyser is parallel to the plane of polarization of the light. In the polar graph, the curve of intensity versus the angle of rotation is a 'eight' 8.

Inference 1:

The electric vector of the incident light is in *one plane* only. Hence, a *plane polarised* light is emerging from the polariser and is incident on the analyser.

PART 2

2.THREE POLAROIDS (P₁, P₂, P₃)

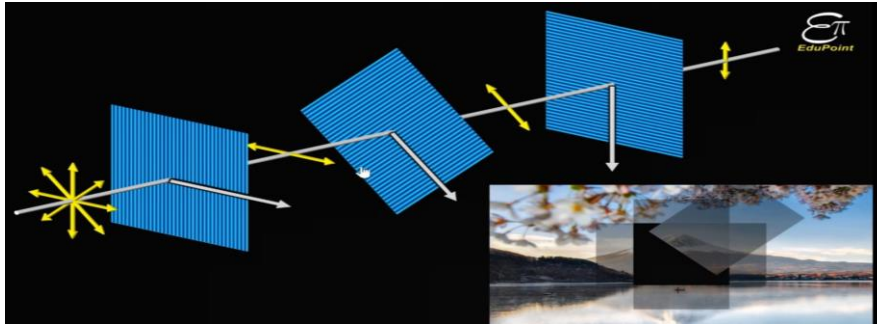


Fig. 6: Three polaroids are in one line. Initial unpolarised light of intensity I_0 has varying intensity after the successive polaroids depending on their orientation with respect to the first polaroid P_1 .

Explanation using Malus' Law.

Formula 2:

$$I_1 = \frac{I_0}{2}$$

$$I_2 = \frac{I_0}{2} \cos^2 \Theta_1$$

$$I_3 = \frac{I_0}{2} \cos^2 \Theta_1 \cos^2 \Theta_2$$

where Θ_1 is the angle between transmission axes of P_1 and P_2 and Θ_2 is the angle between transmission axis of P_2 and transmission axis of analyser P_n ($n = 3$).

Special Case:

For P_1 crossed with P_3 and P_2 removed, the intensity of light through polariser (P_1) along its transmission axis is $I_1 = \frac{I_0}{2}$. After P_3 , all the intensity gets absorbed by the polaroid and the transmission is zero. Under ideal conditions, 0% is transmitted and 100% is absorbed by the crossed polaroid with respect to I_1 .

For P_1 crossed with P_3 and P_2 inserted between them, some intensity is 'restored'!

When $\Theta_1 = 45^\circ$ and $\Theta_2 = (90^\circ - \Theta_1)$,

$$I_1 = \frac{I_0}{2}$$

$$I_2 = \frac{10}{2} \cos^2(45) = \frac{10}{2} \times \frac{1}{2}$$

$$I_3 = \frac{10}{2} \cos^2(45) \cos^2(90-45) = \frac{10}{2} \times \frac{1}{4}$$

Transmission increases to 25% with respect to I_1 .

Under ideal conditions, 75% is absorbed by both P_2 and P_3 with respect to I_1 .

TABLE 2:

Polariser @ 90 degree, Analyser @ 90 degree, 3rd polaroid rotated at an angle θ inbetween analyser and polariser	background current μA	corrected current reading
0	6	2
5	7	3
10	14	12
15	35	31
20	69	65
25	106	102
30	160	156
35	179	175
40	182	187
45	201	197
50	189	185
55	168	169
60	133	129
65	100	96
70	70	66
75	48	42
80	29	25
85	10	6
90	7	3
95	16	12
100	36	32
105	62	58
110	80	76
115	126	122
120	156	152
125	175	171
130	188	184
135	189	185
140	180	176
145	158	154
150	114	110
155	80	76
160	58	52
165	35	31
170	12	8
175	7	3
180	12	8
185	26	22
190	69	61
195	87	83
200	112	108
205	129	125
210	141	137
215	151	147
220	160	156
225	166	162
230	170	166
235	160	156
240	140	136
245	110	106

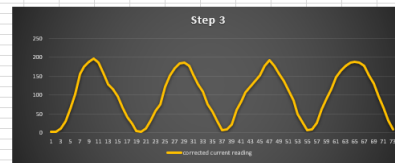
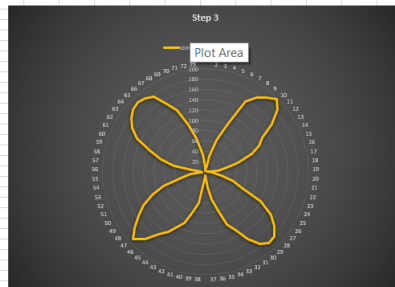


Fig.. 7; (a)(top-plot) Polar graph shows two 'eights' perpendicular to each other. (b) (bottom-plot) Shows four max and four zeros alternately in one complete rotation of the central polaroid.

Result 2:

When a polaroid P_2 is rotated through 360° , in between the crossed polarizer P_1 and analyser P_3 and intensity *varies* across the analyser. There are 4 maxima and 4 minima (zeros) in the intensity versus angle plot.

The polar graph exhibits two 'eights' perpendicular to each other!

Inference 2:

Two plane polarised light are incident on the analyser with *planes perpendicular* to each other. The electric vectors of the incident light are in two perpendicular planes.

3. MORE POLAROIDS

If more polaroids are inserted between polariser and a crossed analyser, what happens?

The Beauty of Malus' Law

Using Malus' Law,

(i) with 8 polaroids between the crossed polariser (P_1) and analyser (P_{10}) at 10° each, the transmission increases to $\sim 76\%$ while absorption is $\sim 24\%$ by the nine polaroids with respect to the first.

(ii) with 88 polaroids between the crossed polariser (P_1) and analyser (P_{90}) at 1° successively, the transmission increases to a whopping $\sim 97\%$ and absorption is only $\sim 3\%$ by the eighty-nine polaroids with respect to the first.

(iii) If there are infinite number of polaroids between the crossed polariser and analyser, what should be the intensity finally? Guess?

For angle in

$\lim_{\theta \rightarrow 0}$ between the successive polaroids, the final intensity after P_∞ should be 100%

Then transmission becomes 100% again, satisfying **Malus' law**, but the electric vector rotates either clockwise or anticlockwise depending upon the rotation of the transmission axis of all the successive polaroids.

An analogy: we observe that a plane polarised light passing through an optically active material emerges with full intensity but rotates clockwise or anticlockwise, depending on whether the optical active medium is laevo- or dextro-rotatory! Examples - Sucrose solution, dextrose solution, chiral liquid crystal, etc.

This is the beauty of the Malus' law!

Formula 3:

$$I_f = I_1 \cos^2\theta_1 \cos^2\theta_2 \dots \dots \dots \cos^2\theta_\infty$$

Where θ_1 is the angle between the transmission axes P_1 and P_2 , θ_2 is the angle between the transmission axes P_2 and P_3 , and so on.

Result 3:

As the number of polaroids between the crossed polaroid increase in number, the intensity of the emergent light increases. And, for infinitely large number of polaroids with inclination is in the limit zero, the emergent light has almost 100% light and the plane is rotated clockwise (anticlockwise) depending on the direction of rotation of the transmission axes of the polaroids.

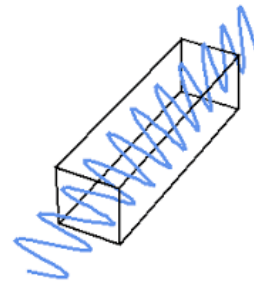


Figure 8: Rotation of the plane of polarization through many polaroids between two crossed polaroids equivalent to optical active media between crossed polaroids.

Inference 3:

The plane of polarization of the linearly polarized light rotates with the rotation of the axes of the transmission of the polaroid and the intensity of the emergent light increases with the number of polaroids.

The net effect of *rotation* and *intensity* of the emergent light through infinitely large number of polaroids is like the *optical active* media.

PART 3

STATES OF POLARISATION

- A retarder can change the state of polarisation.
- A wave plate or retarder is an optical device that alters the polarisation state of a light wave travelling through it. Two common types of wave plates are the half-wave plate and quarter-wave plate. Half-wave plate shifts the polarisation direction of linearly polarised light, as the phase difference is π . The quarter-wave plate converts linearly polarised light into elliptically / circularly / linearly polarised light as the phase difference is $\pi/2$.

Formula 4:

The phase difference between the two electric vectors when emerging through a retarder of length L is

$$\phi = \frac{2\pi}{\lambda_0} \Delta n L$$

For a Quarter Wave Plate, the path difference is $\frac{\lambda}{4}$, the phase difference between ordinary ray and extraordinary ray is $\frac{\pi}{2}$.

Formula 5:

The intensity after traveling the QWP and the crossed polaroids is

$$I_f = a^2 \cos^2 \alpha + b^2 \sin^2 \alpha$$

where Θ is the angle between the electric vector E (or transmission axis of the polarizer) incident on the QWP and its optic axis and α is the angle between the optic axis of the QWP and the transmission axis of the analyser. And the emergent light is elliptically polarised generally.

$a = E \cos \Theta$ (amplitude of the electric vector along the optic axis of the QWP = major axis of the ellipse)

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$b = E \sin \Theta$ (amplitude of the electric vector perpendicular to the optic axis of the QWP = minor axis of the ellipse)

Special cases:

$a = 1, b = 0$

OR

$a = 0, b = 1$ the emergent light is linearly polarised

$a = b$, the emergent light is circularly polarised light

$a \neq b$ the emergent light is elliptically polarised light.

a and b depend on Θ .

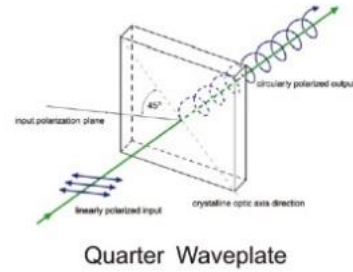


Figure 9: when the incident plane polarised light is incident at 45° to the Optic axis of the QWP, the emergent light is circularly polarised

TABLE 3.

Polariser at an angle-80
Analyser rotated
QWP at an angle 20 background current = 5µA

Angle Θ of the rotated QWP kept inbetween polariser and analyser	current in μA	corrected current reading in μA
0	227	2.22E+02
5	211	2.06E+02
10	177	1.72E+02
15	135	1.30E+02
20	114	1.09E+02
25	80	7.50E+01
30	63	5.80E+01
35	47	4.20E+01
40	29	2.40E+01
45	27	2.20E+01
50	30	2.50E+01
55	39	3.40E+01
60	56	5.10E+01
65	70	6.50E+01
70	86	8.10E+01
75	108	1.03E+02
80	133	1.28E+02
85	157	1.52E+02
90	184	1.79E+02
95	225	2.25E+02

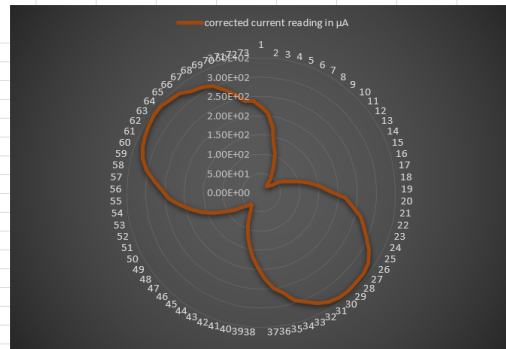


Fig 10: QWP rotated to 20° , the plot shows an elliptically polarised light and a is the major axis and b the minor axis. For one rotation of the analyser, the intensity changes from maxima to a minima (not zero), twice.

TABLE 4

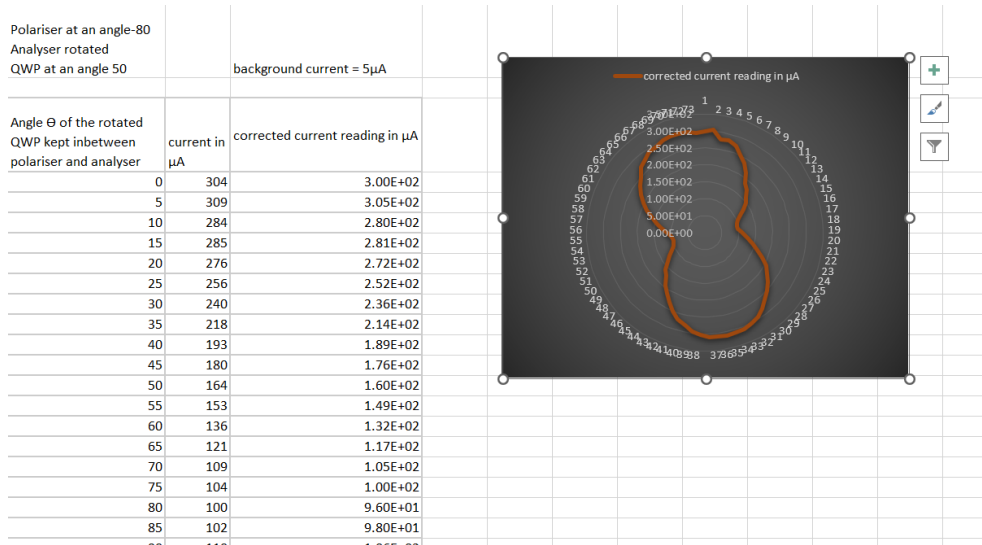


Fig 11: QWP rotated to 50° the plot shows an elliptically polarised light with greater ellipticity.

TABLE 5

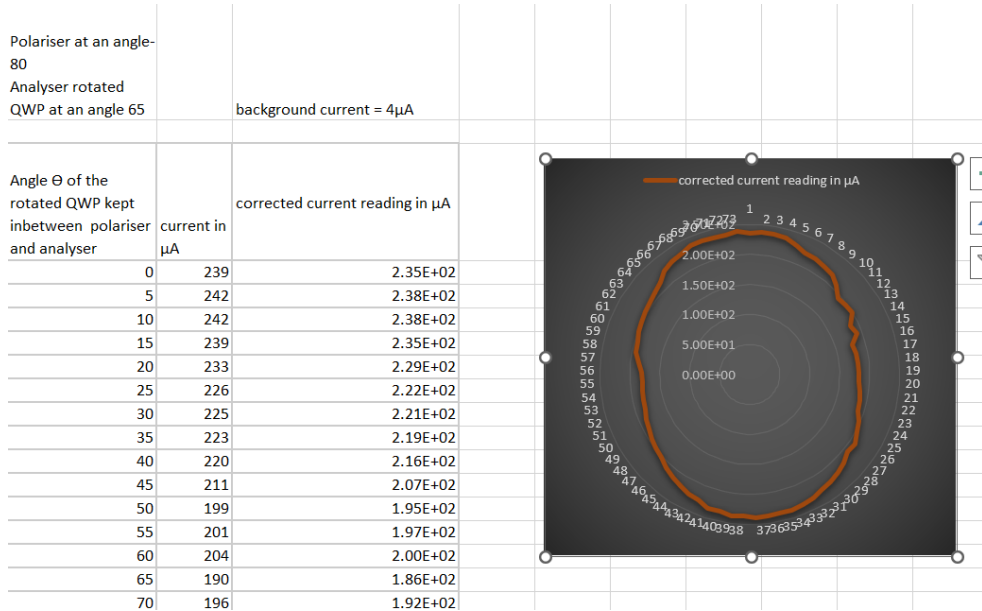


Fig 12: QWP rotated to 65°, the plot shows a nearly circularly polarised light.

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TABLE 6

Polariser at an angle-80		background current = 4 μ A	
Analyser rotated			
QWP at an angle 80			
Angle Θ of the rotated QWP kept inbetween polariser and analyser	current in μ A	corrected current reading in μ A	
0	172	1.68E+02	
5	163	1.59E+02	
10	154	1.50E+02	
15	148	1.44E+02	
20	140	1.36E+02	
25	136	1.32E+02	
30	138	1.34E+02	
35	143	1.39E+02	
40	144	1.40E+02	
45	154	1.50E+02	
50	156	1.52E+02	
55	170	1.66E+02	
60	184	1.80E+02	
65	194	1.90E+02	
70	208	2.04E+02	
75	220	2.16E+02	
80	230	2.26E+02	
85	235	2.31E+02	
90	243	2.39E+02	
95	254	2.50E+02	
100	260	2.56E+02	
105	266	2.62E+02	
110	270	2.66E+02	
115	272	2.68E+02	

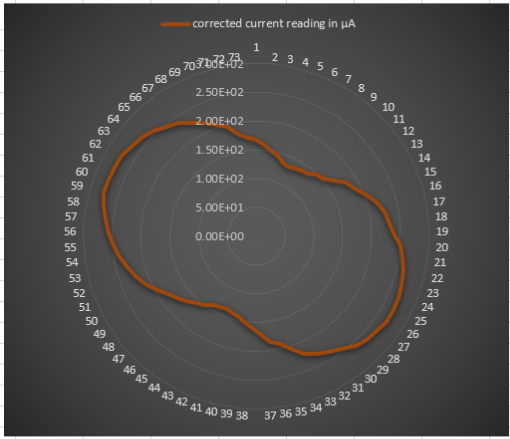


Fig 13: QWP rotated at an angle 80°, the plot shows an elliptically polarised light with $a < b$.

TABLE 7

Polariser at an angle-80		background current = 4 μ A	
Analyser rotated			
QWP at an angle 95			
Angle Θ of the rotated QWP kept inbetween polariser and analyser	current in μ A	corrected current reading in μ A	
0	191	1.87E+02	
5	170	1.66E+02	
10	145	1.41E+02	
15	127	1.23E+02	
20	105	1.01E+02	
25	90	8.60E+01	
30	73	6.90E+01	
35	60	5.60E+01	
40	55	5.10E+01	
45	58	5.40E+01	
50	66	6.20E+01	
55	76	7.20E+01	
60	100	9.60E+01	
65	110	1.06E+02	
70	133	1.29E+02	
75	166	1.62E+02	
80	182	1.78E+02	
85	204	2.00E+02	
90	223	2.19E+02	
95	247	2.43E+02	
100	265	2.61E+02	
105	282	2.78E+02	
110	296	2.92E+02	
115	311	3.07E+02	
120	319	3.15E+02	

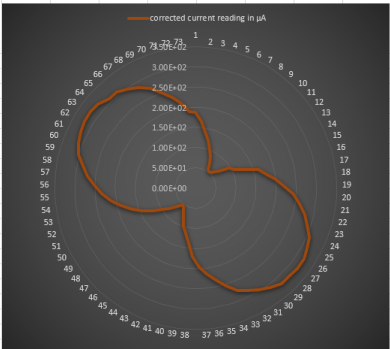


Fig 14: QWP rotated to 95°, shows an elliptically light with decreasing a .

Result 4:

A retarder introducing a phase difference of $\pi/2$ (Quarter Wave Plate) in the plane polarised light produces an elliptically polarised light. The ellipticity gradually increases with the angle Θ between the plane of polarization and the optic axis of the QWP plate.

For $\Theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$, the emergent light is plane polarised.

For $\Theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$, the emergent light is circularly polarised.

When Θ increases from 0° to 45° , the major axis decreases, and the minor axis increases.

When Θ increases from 45° to 90° , the major axis increases and the minor axis decreases.

Inference 4:

A QWP in the path of a *plane* polarised light produces *elliptically* polarised light in general. For specific angles, the emergent light is plane polarised or circularly polarised.

Part 4

QUARTER WAVE PLATE ROTATED BETWEEN POLARISER AND ANALYSER

TABLE 8:

Polariser at an angle-80 Analyser at an Angle-45 QWP rotated		background current = 6 μ A	
Angle Θ of the rotated QWP kept inbetween polariser and analyser	current in μ A	current	corrected current reading in μ A
0	118		1.12E+02
5	145		1.39E+02
10	165		1.59E+02
15	178		1.72E+02
20	188		1.82E+02
25	186		1.80E+02
30	180		1.74E+02
35	164		1.58E+02
40	134		1.28E+02
45	101		9.50E+01
50	68		6.20E+01
55	40		3.40E+01
60	26		2.00E+01
65	21		1.50E+01
70	30		2.40E+01
75	66		6.00E+01
80	100		9.40E+01
85	136		1.30E+02
90	162		1.56E+02
95	182		1.76E+02
100	194		1.88E+02
105	194		1.88E+02
110	175		1.69E+02
115	158		1.52E+02
120	133		1.27E+02
125	100		9.40E+01
130	68		6.20E+01
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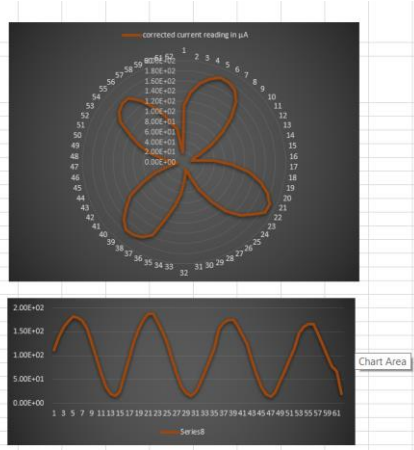


Fig 15: The plot of intensity versus angle shows four maxima and four minima in one rotation. Here the QWP is rotated between the polariser and the analyser.

COMPARISON OF PART 2 AND PART 4

Result 5: The final intensity through a quarter wave plate is doubly intense compared to the final intensity through 3 polaroids, keeping the position of the lobes at the same angle.

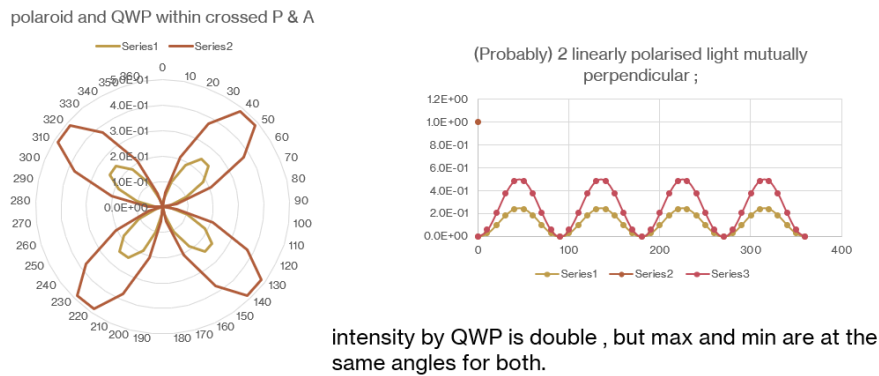


Fig 16: intensity variation due to 3 polaroids and a QWP placed between crossed polaroids are similar, except that the intensity through the QWP is double.

Inference 5:

Malus's law explains the rotation and the intensity variation of light under all situations. That is the beauty of this entire experiment!

Conclusion:

A simple Malus' Law can explain different aspects of Polarization, so well-

- The intensity of a plane polarized light on emerging a polaroid varies according to the *cos square rule* -THE MALUS' LAW.
- The intensity variation is a confirmatory test for a plane polarized light *max-zero-max-zero* or the plot of '*eight*' in a polar graph.
- The intensity variation *differs* when the number of polaroids is *increased* in the path of a plane polarised light.
- Malus' law also explains the *increase* of the transmission light with *increase* of the number of the polaroids between the crossed polaroids.
- The *rotation* of the plane of polarization along with the rotation of the polaroids is *implicit* in the Malus' Law

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- Malus's Law *for infinite polaroid* explains the rotation and intensity of light through an *optically active medium*.
- Malus' Law suggests the presence of two plane polarised perpendicular light by max-zero-max-zero-max-zero-max-zero or *two 'eights'* perpendicular to each other.
- Plane polarized Light through a QWP emerges with a phase shift. Malus' Law explains linearly, circularly, and elliptically light.

This is the BEAUTY of simple MALUS' LAW!