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Black Hole Radiation by Tunnelling Formulation

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Abstract. In this paper, we illustrate the Hawking radiation as a tunneling phenomenon in the Hamilton-Jacobi, using WKB approximation and other methods radiation from non-singular. Space –time diagram of the Krushkal and Painleve Gullstrand coordinates are being sown to acknowledge their regular behaviour at the horizon.

Keywords: Principal value, Killing vector, Halmilton-Jacobin equation, Regular coordinates.
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1. Introduction

In 1974, Hawking discovered theoretically that black holes emit radiation and the radiation is a thermal radiation which is due to the temperature of black holes. He had derived the expression for temperature using the quantum field theory in curved space-time for Schwarzschilld like black holes and the formula given was $k/2\pi$ where *k* is surface gravity [1]. Several other techniques were developed along the line to explain this phenomenon of emission. Presently, the tunneling method is more useful and easy facilitated by complex path analysis [2]. In this method we have a scalar particle in the black hole background which tunnel through the black hole horizon considered as a barrier. It is a relativistic massless particle described by Klein Gordon equation

$$\frac{h^2}{(-g)^{1/2}}\partial_u (g^{uv}(-g)^{1/2}\partial_{v\phi}) = 0 \tag{1}$$

In WKB approximation one can write

$$\phi = \exp\left(-\frac{i}{h}S + \text{L}\right) \tag{2}$$

Substitute ϕ in Eqn.(1) to obtain the Hamilton- Jacobi equation

$$g^{\mu\nu}\partial_{\mu}S\,\partial\mu S=0 \tag{3}$$

2. Schwarzschild Spacetime

2.1 *Regular Coordinate*

The coordinates which are well behaved and free from coordinate singularities.

2.1.1 Krushkal coordinate

$$ds^2=\frac{4r_h^3}{r}c^{\frac{v}{r}h}\left(-dT^2+dR^2\right)+r^2d\Omega^2 \tag{4}$$

where $d\Omega^2=d\theta^2+\sin^2\theta\phi^2$. Now transform this matric by using X=R-R.

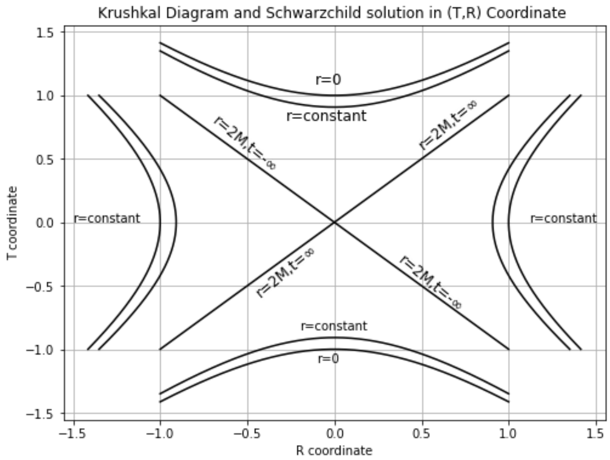

Fig. 1: Krushkal diagram

Y = R+R transformation to obtain metric of the form

$$ds^2=-\frac{4r_h^3}{r}e^{-\frac{r}{r_h}}dXdY+r^2d\Omega^2 \tag{5}$$

The Hamilton- Jacobi (H- J) equation in the case of above metric for spherically symmetric S is

$$\frac{\partial S}{\partial X}\frac{\partial S}{\partial Y}=0\ . \tag{6}$$

S can have solutions one depending upon X (outgoing) and other depending upon Y (ingoing)

$$S_{out}=-\frac{E}{k}\int\frac{dX}{X},\ S_{in}=\frac{E}{k}\int\frac{dY}{Y}. \tag{7}$$

By solving the X integral to obtain the imaginary factor $i\pi$ which is necessary to produce the Hawking temperature $\frac{hk}{2\pi}$ for emission.

2.1.2 *Painleve and Gullstrand coordinates*

The metric is given [3]

$$ds^2 = -\left(1-\frac{2M}{r}\right)dT^2 + 2\sqrt{\frac{2M}{r}}\,dT dr + dr^2 + r^2 d\Omega^2 . \qquad (8)$$

The H-J equation in the above metric for spherically symmetric *S* is

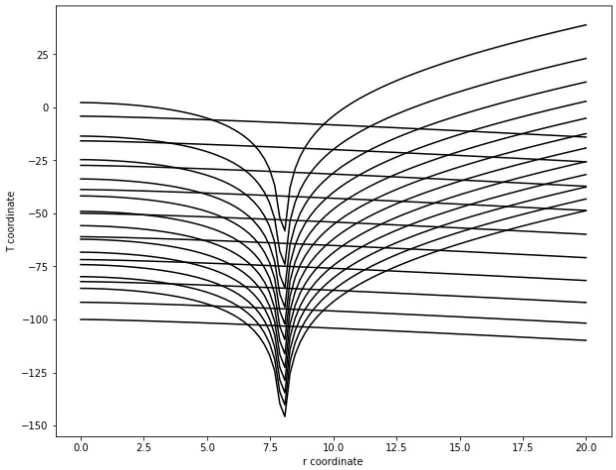

Fig. 2: Painleve-Gullstrand space-time diagram

$$\left(\frac{dS}{\partial T}\right)^2 + 2\sqrt{\frac{2M}{r}}\left(\frac{\partial S}{\partial T}\right)\left(\frac{\partial S}{\partial r}\right) + \left(1-\frac{2M}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2 = 0 \qquad (9)$$

Owing to the killing vector is ∂_T we can use the concept that $S = ET + S_0(r)$. Solving for $S_0(r)$ by the quadratic method to obtain the expression of the kind

$$S_0(r) = E\int^r dr \frac{\pm 1 - \sqrt{\frac{2M}{r}}}{1-\frac{2M}{r}} .$$

On solving the above integral and substituting the radially dependent part of action into the total expression for action we obtain two solutions

r

$$S_{out} = ET - E\left[4m\pi i + P\cdot V\int^{T} r\left(\frac{1+\sqrt{\frac{2M}{r}}}{r-2M}dr\right)\right] \quad (10)$$

$$S_{in} = ET + E + \left(P\cdot V\int^{T} r\frac{1+\sqrt{\frac{2M}{r}}}{r-2M}dr\right) \quad (11)$$

Here P.V represents principal value. The outgoing solution contains imaginary term which is used to reproduce Hawking temperature of $\frac{h}{8\pi M}$.

3. Results

The standard Hawking temperature obtained in both coordinates is $\frac{hk}{2\pi}$.

4. Conclusion

1. In the preceding work, we have examined the Hawking radiation as tunnelling from horizon under the framework of regular coordinates, resulting in two distinct solutions. Specifically, the outgoing solution manifests an imaginary term, while the incoming solution lacks such a term. This highlights that emission is limited by a factor while absorption is absolute. However, in the case of Krushkal metric, both solutions encompass an imaginary term due to simultaneous inclusion of black hole and white hole region in the metric. Notably, if we solely focus on the black hole portion, where the parameter Y is constant, we still arrive at the conventional Hawking temperature.
2. The emergence of standard Hawking temperature in these coordinates, devoid of any reliance on boundary conditions, signifies the efficacy of regular coordinates for investigating the Hawking effect.

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