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Model of Contrast Sensitivity Function in Patients with Age-Related Macular Degeneration

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Abstract. Using the Stiles-Crawford function, we have calculated the relation between the contrast sensitivity of the human eye for a grating and the spatial frequency of the grating. Using the fact that at spatial frequency 50 cycles/degree of the grating, the value of contrast sensitivity is 1, the value of contrast sensitivity at any other spatial frequency is calculated using the assumption that the minimum number of photons required to elicit the visual response in the retina is constant. Again, we assume that taking width of the grating unity when we increase the area, the length should increase. An increase in length means a decrease in spatial frequency, so the intensity or contrast decreases and the contrast sensitivity value increases. Again, using the same Stiles Crawford function but now with a different directionality constant for AMD, we have obtained the age-related macular degeneration curve. Here the value of the directionality constant for AMD is found to be$0.041/\mathrm{mm}^2$. It is seen that for a particular spatial frequency, the contrast sensitivity of the healthy eye is greater than that of the AMD eye, and this difference increases with a decrease in the spatial frequency of the grating.

1. Introduction

Stiles Crawford Effect

Stiles and Crawford discovered in 1933 that light entering the human eye at the centre of the pupil is several times more sensitive in producing the sensation of vision than light entering through the periphery in an aberration-free optical system. This effect is a manifestation of the directional nature of photoreceptors, especially the cones. The effect is mathematically expressed as [1]

$$\eta_r = \eta_0 e^{-\rho r^2} \tag{1}$$

Where $\eta_r =$ intensity of light coming at a distance r from the centre of the pupil

η_0 = intensity of light coming through the centre of the pupil

ρ = directionality constant of the retina

r = light entrance distance from pupil centre

The value of the directionality constant for a healthy eye is given as

$\rho = 0.115/\mathrm{mm}^2$ [2].

Contrast is the difference in luminance that makes an object distinguishable. Sinusoidal gratings allow further characterization of the visual system's sensitivity to spatial changes in luminance. From Schwartz, an optional way to define the contrast of a grating is

$$\text{Contrast} = \frac{\Delta L}{L_{avg}}$$

Where ΔL is the difference between peak and average luminance, andL_{avg} is the average luminance of the peak and trough. Michaelson contrast is defined as

$$(L_{max} - L_{min})/(L_{max} + L_{min})$$

Where L_{max} = maximum luminance of the grating

L_{min} = minimum luminance of the grating

The contrast threshold can be defined as the minimum contrast that can be resolved by the patient. Contrast sensitivity is the inverse of contrast threshold [3]. Or,

$$\text{Contrast Sensitivity(C. S)} = \frac{1}{\text{Contrast threshold}} \qquad (2)$$

The graph between contrast sensitivity function and spatial frequency shows that for higher spatial frequency, the value of the contrast sensitivity function, which indicates the rise in minimum contrast required to resolve the grating at this higher frequency.

Age-related macular degeneration (AMD) is a disease primarily affecting the macula and an increasingly prevalent cause of irreversible blindness in the industrialised world. A common early symptom in macular degeneration is the perception of blurring and metamorphosia in the central visual field, with attendant reduced visual acuity [4]. Here, we would like to propose that the pathological mechanism in at least some forms of MD involves a disturbance in the mechanism of photoreceptor alignment. Due to this disturbance, the centre-to-centre spacing of the cone mosaic is increased, and the visual acuity is reduced.

Here we have assumed that this reduction in visual acuity leads to a decrease in the resolving power of the grating for a particular contrast. Having defined the definition of contrast and taking the average of maximum and minimum intensity to be 1, we may say alternatively that contrast and intensity can be taken as the same. So due to misalignment of the photoreceptor cone, the AMD-affected patient cannot resolve the grating of a particular spatial frequency, which can be resolved in a healthy eye. So, to resolve it, the intensity or, alternatively, the contrast threshold should be increased. In other words, for an AMD-affected eye, the contrast sensitivity for a particular spatial frequency decrease compared to its value in a healthy eye for the same spatial frequency, which we want to show in this paper.

2. Methodology

The length of an object that subtends an angle of *1 degree* at the retina$l = r \times \frac{\pi}{180}$

Here r =distance of the pupil from the retina $= 22.2\text{mm}$ [5]

So, $l = 22.2 \times \frac{\pi}{180} = 0.387\text{mm}$

For a spatial frequency of 2 cycles/degree, there are two cycles in one degree, so

$$\text{One cycle length} = \frac{\text{length of 1 degree}}{2} = \frac{0.387\text{mm}}{2} = 0.193\text{mm}$$

This cycle now has a brighter and darker side. So, the brighter half cycle has length $\frac{0.193}{2}\text{mm}$. Again, this half cycle remains at the middle of the pupil. So, half of this having length $\frac{0.193}{4}\text{mm}$ remains at one side while the other half remains at other side. Now, the intensity is seen at the retina according to the Stiles Crawford Effect.

Again, this half-cycle remains in the middle of the pupil. So, half of this $\frac{0.193}{4}\text{mm}$ length remains on one side, while the other half remains on the other. Now the intensity is seen at the retina, according to the Stiles Crawford Effect. So, the total number of photons for a half cycle $= \eta_0 \int_{-\frac{0.193}{4}}^{\frac{0.193}{4}} e^{-.230\times(x^2)} dx$

Using Simpson's 1/3rd rule [6], the above integral becomes

$$\eta_0 \frac{0.368}{3 \times 4}\left[2 + e^{-0.230\times\left(\frac{0.193}{4}\right)^2}\right]$$

$$\cong \eta_0 \times \frac{0.368}{4}$$

For 50 cycles/degree of spatial frequency,

one cycle length is $\frac{0.193}{25}$ mm

The total number of photons

$$= \text{Intensity}\left(\text{at } 50\ \frac{\text{cycles}}{\text{degree}}\right) \times \text{Area} = \int_{-\frac{0.193}{100}}^{\frac{0.193}{100}} e^{-.115\times(x^2)} \times e^{-.115\times(x^2)} dx$$

$$\int_{-\frac{0.193}{100}}^{\frac{0.193}{100}} e^{-.230\times(x^2)} dx$$

$$\cong \int_{-0.002}^{0.002} e^{-.230x^2}\, dx$$

$$\int_{-0.002}^{0.002} e^{-.230x^2}\, dx = \frac{0.004}{3}\left[2 + e^{-0.230(0.002)^2}\right] = \frac{0.004}{3} \times 2.999$$

$$= 0.999 \times 0.004 = 0.0039$$

So, we get the total number of photons $= 0.0039$

Equating both we get

$$\eta_0 \times \frac{0.386}{4} = 0.0039$$

$$\Rightarrow \frac{1}{\eta_0} = \frac{386}{4 \times 3.9} = 24.743$$

Now for $m \frac{\text{cycles}}{\text{degree}}$ spatial frequency, one cycle length is $\frac{0.193\times 2}{m}$

Then the total number of photons

$$= \eta_0 \times \int_{-\frac{0.193}{2m}}^{\frac{0.193}{2m}} e^{-.115x^2} \times e^{-.115\times(x^2)}\, dx = \eta_0 \times \int_{-\frac{0.193}{2m}}^{\frac{0.193}{2m}} e^{-.230x^2}\, dx$$

Using Simpson's 1/3rd rule [6]

$$= \eta_0 \times \left(\frac{0.193}{3m}\right)\left[2 + e^{-0.230\times\left(\frac{0.193}{2m}\right)^2}\right]$$

Then equating this to the number of photons for a spatial frequency of 50 cycles/degree

$$\eta_0 \times \left(\frac{0.193}{3m}\right)\left[2 + e^{-0.230\times\left(\frac{0.193}{2m}\right)^2}\right] = 0.0039 \qquad (3)$$

Then for spatial frequency m of the grating, the contrast sensitivity at the centre of the pupil is

$$\eta_0 = \frac{0.0039}{\left(\frac{0.193}{3m}\right)\left[2 + e^{-0.230\times\left(\frac{0.193}{2m}\right)^2}\right]} \Rightarrow C.S.F.$$

$$= \frac{193}{3 \times 3.9} \times \frac{1}{m} \times \left[2 + e^{-0.230\times\left(\frac{0.193}{2m}\right)^2}\right]$$

$$= \frac{16.496}{m}\left[2 + e^{-\frac{0.002}{m^2}}\right] \tag{4}$$

Then, minimum intensity or contrast occurs at the edge, which is given by

$$\eta = \eta_0 \times e^{-0.115\times\left(\frac{0.193}{2m}\right)^2} = \frac{0.0039 \times e^{-0.115\times\left(\frac{0.193}{2m}\right)^2}}{\left(\frac{0.193}{3m}\right)\left[2 + e^{-0.230\times\left(\frac{0.193}{2m}\right)^2}\right]}$$

$$\frac{1}{\eta} = \frac{1}{\eta_0 \times e^{-0.115\times\left(\frac{0.193}{2m}\right)^2}} = \frac{16.496}{m}\left[2 + e^{-\frac{0.002}{m^2}}\right] \times e^{\frac{0.001}{m^2}} \tag{5}$$

Now we wish to obtain a similar formula for contrast sensitivity for an eye affected by age-related macular degeneration. For this, we need the directionality constant of the Stiles Crawford Effect in the AMD eye. As the directionality constant decreases almost to zero in an AMD affected eye, we have taken$\rho = 0.041/\mathrm{mm}^2$.

From the equation of AMD and the graphs obtained, it was found that, the Stiles-Crawford effect directionality constant for age-related macular degeneration is taken here $0.041/\mathrm{mm}^2$ For an AMD-affected eye, the total number of photons for spatial frequency 50 cycles/degree is

$$= \int_{-\frac{0.193}{100}}^{\frac{0.193}{100}} e^{-.041x^2} \times e^{-.041x^2}\, dx$$

$$\cong \int_{-0.002}^{0.002} e^{-.082x^2}\, dx$$

Considering the integral

$$\int_{-0.002}^{0.002} e^{-.082x^2}\, dx = \frac{0.004}{3}\left[2 + e^{-0.082\times(0.002)^2}\right] = \frac{0.004}{3} \times 2.999 = 0.004$$

total number of photons for spatial frequency= m cycles/degree

$$= \eta_0 \times \int_{-\frac{0.193}{2m}}^{\frac{0.193}{2m}} e^{-.041x^2} \times e^{-.041x^2}\, dx$$

Considering the integral

$$\int_{-\frac{0.193}{2m}}^{\frac{0.193}{2m}} e^{-.082x^2}\, dx = \left(\frac{0.193}{3m}\right)\left[2 + e^{-0.082\times\left(\frac{0.193}{2m}\right)^2}\right]$$

Then the total number of photons

$$\eta_0 \times \left(\frac{0.193}{3m}\right)\left[2 + e^{-0.082\times\left(\frac{0.193}{2m}\right)^2}\right]$$

Equating both

$$\eta_0 \times \left(\frac{0.193}{3m}\right)\left[2 + e^{-0.082\times\left(\frac{0.193}{2m}\right)^2}\right] = 0.004$$

$$\Rightarrow \frac{1}{\eta_0} = \frac{\left(\frac{0.193}{3m}\right)\left[2 + e^{-0.082\times\left(\frac{0.193}{2m}\right)^2}\right]}{0.004}$$

$$\Rightarrow \frac{1}{\eta} = \frac{1}{\eta_0 \times e^{-0.041\times\left(\frac{0.193}{2m}\right)^2}} = \frac{16.083}{m}\left[2 + e^{-\frac{0.00075}{m^2}}\right] \times e^{0.041\times\left(\frac{0.193}{2m}\right)^2} = \frac{16.083}{m}\left[2 + e^{-\frac{0.00075}{m^2}}\right] \times e^{\frac{0.000375}{m^2}} \quad (6)$$

4. Results and Discussion

1. A graph is plotted by taking spatial frequency along the x-axis and the contrast sensitivity function along the y-axis, as shown in Figure 1. It is found that, with decreasing spatial frequency, the contrast sensitivity function increases. If we take the minimum number of photons required to produce a visual response to be constant. Then, by increasing area, the intensity or contrast decreases, so the reciprocal of this increases. The reciprocal is called the contrast sensitivity function in the graph.

2. A portion of Figure 1's spatial frequencies between 0.25 cycles/degree and 0.26 cycles/degree is shown in Figure 2. Also, another curve is shown in the figure, which shows the contrast sensitivity function for an eye with age-related macular degeneration. Visual acuity suffers as a result of misalignment. This means it cannot resolve the same grating

that a healthy eye can. To resolve it, the intensity of light must be increased. In other words, at the same spatial frequency, the reciprocal of contrast threshold, or contrast sensitivity, is lower in an AMD eye than in a healthy eye.

3. In figure 3, the contrast sensitivity function at higher spatial frequencies (50 cycles per degree to 55 cycles per degree) is plotted. Figure 2 shows that the difference in CSF between a healthy eye and an AMD eye is greater in Figure 2 and smaller in Figure 3.

4. Difference between C.S.F. of healthy eye and C.S.F. for AMD eye is plotted in Figure 4 against spatial frequency. It is seen that the difference in C.S.F. between healthy and AMD eye increases with decrease in spatial frequency which is in agreement with the data available [11].

The difference in contrast sensitivity between healthy and AMD affected eye is given by

$$\frac{16.496}{m}\left[2+e^{-\frac{0.002}{m^2}}\right]\times e^{\frac{0.001}{m^2}}-\frac{16.083}{m}\left[2+e^{-\frac{0.00075}{m^2}}\right]\times e^{\frac{0.000375}{m^2}}$$

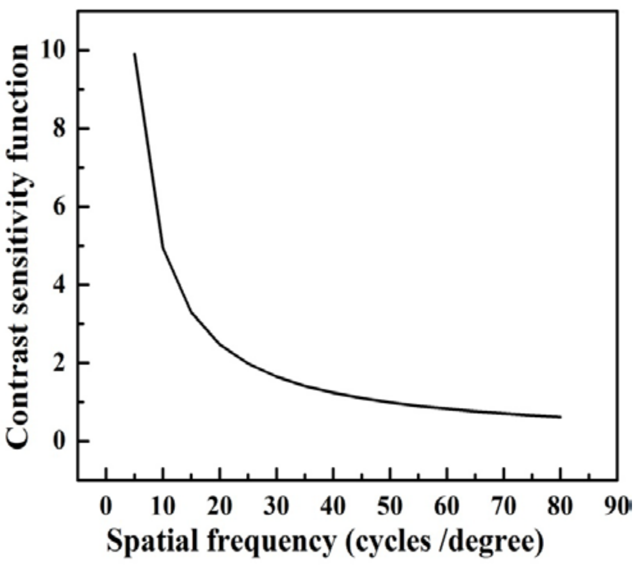

Fig. 1: For a healthy eye, the contrast sensitivity of the retina is a function of the spatial frequency of the grating.

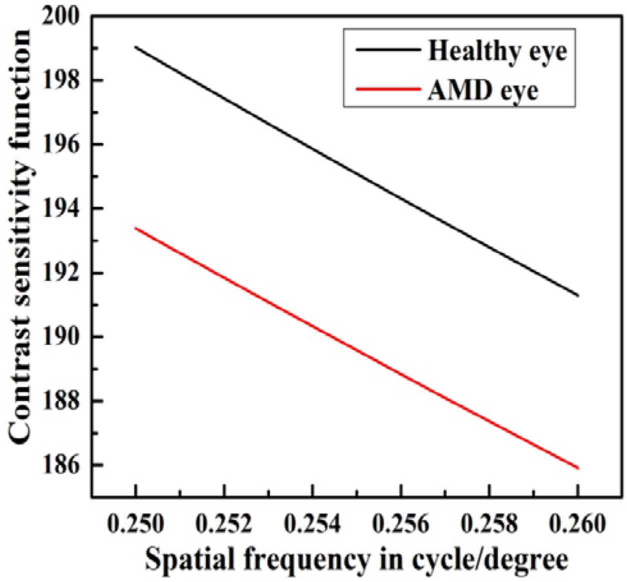

Fig. 2: The contrast sensitivity of an AMD eye is less than that of a healthy eye.

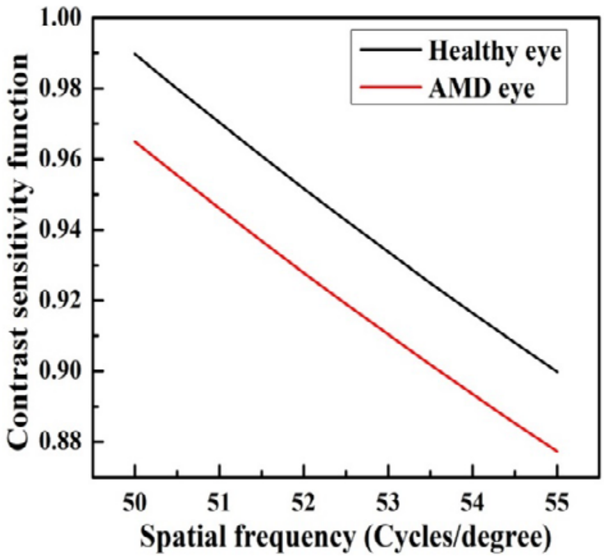

Fig. 3: At higher spatial frequencies, there is less difference between the contrast sensitivity of a healthy eye and that of an AMD eye.

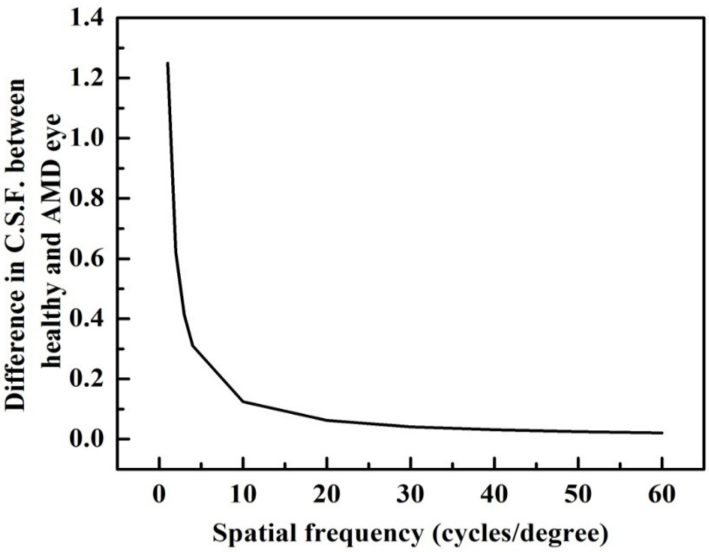

Fig. 4: Figure depicts an increase in contrast sensitivity of the eye, first slowly and then rapidly, as the spatial frequency of the grating decreases.

5. Conclusion

The formula for the directionality constant for a healthy eye can be derived in a simple way from the idea that the minimum number of photons required to elicit a visual response is constant for a healthy eye. The importance of the method is the use of the Stiles-Crawford function in deriving the formula for CSF. This is because by putting the directionality constant for age-related macular degeneration, the contrast sensitivity function in that disease can be derived. For patients with AMD, the loss of contrast sensitivity compared to a healthy eye is more pronounced in the lower spatial frequency range.

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