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Effect of Gravitational Correction in a Supersymmetric E_6 grand Unified Theory

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Abstract: We consider a supersymmetric E_6 **grand unified theory** (GUT) in presence of non-renormalizable dimension-5 operator, which induces gravitational correction. The present model allows intermediate D-parity violating trinification symmetry $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ with asymmetric $SU(3)_L$ and $SU(3)_R$ coupling. It is observed that unification mass scale M_U and inverse GUT coupling constant α_G^{-1} remain unaffected by the gravitational correction, whereas the electroweak mixing angle $\sin^2\theta_W$ is influenced by it. The nice feature of the present work shows that, inclusion of the gravitational correction, permits low intermediate scale, accessible to experimental detection as well as with admissible unification mass in comply with experimental proton decay constraint.

Keywords: Supersymmetry, E_6 GUT, Trinification symmetry, D-parity, Gravitational correction, Proton decay.

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1. Introduction

Grand unified theory (GUT) coupled with supersymmetry (SUSY) [1] has long been considered to be a very attractive paradigm for physics beyond the Standard Model (SM). It naturally addresses some of the crucial issues of SM, i.e. the gauge hierarchy problem, stabilization of the electroweak scale against quantum corrections etc. The local version of SUSY automatically provides a gravitational interaction, paving a road for the complete unification of all forces in nature. The experimental detection of SUSY GUTs at the Large Hadron Collider (LHC) and the future International Linear Collider (ILC) has been a challenge for experimentalists. Keeping in view of the relevance of supersymmetric theory, the present study focuses on the viability of the exceptional group E_6 [2] coupled with supersymmetry, as the grand unified theory. With the prevailing interest in the superstring theory, exceptional gauge models as the unified GUT, are highly acknowledged by the particle physicists. E_6 as obtained due to spontaneous compactification of E_8 superstring theory, seems to play a crucial role for observable low energy phenomenology.

In this paper, we investigate the phenomenological impact of gravitational correction mediated through non-renormalisable dimension-5 operator [3] in the E_6 SUSY GUT model via its maximal subgroup $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ [4], as the intermediate symmetry. The inclusion of this operator in the Lagrangian, actually points out the memory of Planck scale physics, which in turn imply the unification of the GUT model with gravity. It has significant effect on the coupling strengths of the interactions. In the present case, by fine-tuning the gravitational correction parameter, the intermediate mass scale can be lowered, so that one can have viable phenomenology, accessible to experimental detection.

The present paper contains the following sections. The general framework for the model is given in section-2. In the next section we show the evolution equations of gauge couplings and model prediction for the unification mass, inverse GUT coupling constant and electroweak mixing angle have been discussed. The last Section is devoted to a discussion on the phenomenological implications of the model.

2. The model framework

The model framework is based on the following symmetry breaking chain from E_6 GUT to $SU(3)_C \otimes U(1)_Q$ via intermediate trinification gauge symmetry:

$$E_6 \otimes SUSY \overset{M_U}{\rightarrow} SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \ (g_{3L} \neq g_{3R})(\mathbb{G}_{333}) \otimes SUSY$$

$$\overset{M_I}{\rightarrow} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \ (\mathbb{G}_{321}) \otimes SUSY$$

$$\overset{M_Z}{\rightarrow} SU(3)_C \otimes U(1)_Q (\mathbb{G}_{31}) \quad (1)$$

The model utilises the Higgs $\{650_H \oplus (27_H \oplus \overline{27}_H)\}$ belonging to E_6 for symmetry breaking purpose. Here unlike the non-SUSY case, the models are constructed to include conjugate representation $\overline{27}_H$ for *D*-flatness condition so that *D*-term of the potential vanishes. As per the breaking channel, E_6 breaks to the $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ symmetry at the unification scale M_U when the parity odd singlet $(1,1,1) \subset 650_H \subset E_6$ gets vacuum expectation value (VeV). Due to the VeV of parity odd singlet, left-right discrete symmetry (D-parity) is violated such that we have asymmetric gauge couplings for $SU(3)_L$ and $SU(3)_R$ $i.e.$ $g_{3L} \neq g_{3R}$. Then the D-parity violating trinification gauge symmetry breaks to the Standard Model at the intermediate mass scale M_I when the fields $(1,\overline{3},3) \oplus (1,3,\overline{3}) \subset 27_H \oplus \overline{27}_H$ get non-zero vacuum expectation value. The last stage of symmetry breaking $\mathbb{G}_{321}$ to $\mathbb{G}_{31}$ is done by the conventional weak doublets $(1,2,1/2)_{27} \oplus (1,2,-1/2)_{\overline{27}}$ at M_Z scale reproducing all known SM fermion masses. For simplicity, we assume that the SUSY scale M_S coincides with M_Z, i.e. the supersymmetry is broken at the M_Z scale.

It is observed that [5, 6] with only $(1,\overline{3},3) \oplus (1,3,\overline{3}) \subset 27_H \oplus \overline{27}_H$ the unification mass scale M_U and the intermediate mass scale M_I are nearer to each other (>10^{15} GeV). Since GUT model with high intermediate scale has the disadvantage that, the neutrino seesaw related new physics is well hidden form low energy and collider probes thereby making it untestable. Hence in the present study our focus is to obtain i) low M_I to be experimentally accessible and ii) to have admissible value of M_U satisfying proton decay constraint. We aim to achieve this through the gravitational correction as well as in presence of some additional light particles. We now give a brief discussion on both options.

Gravitational correction: As mentioned before, gravitational correction is mediated through non-renormalisable dimension-5 operator in the Lagrangian, given as:

$$\mathbb{L}_{NRO} = -\frac{\eta}{4M_G}\mathrm{Tr}\big(F_{\mu\nu}\Phi_{650}F^{\mu\nu}\big) \tag{2}$$

which is operative at the unification mass scale. Here $M_G \leq M_{Pl}$, the Planck scale $\simeq 10^{19}$ GeV, η is a dimensionless parameter and $F_{\mu\nu}$ is the field strength tensor. As has been mentioned before, Φ_{650} is the odd singlet scalar which takes the VEV

$$\langle\Phi_{650}\rangle = \frac{\langle\phi^0\rangle}{\sqrt{6}}\mathrm{diag}\left\{\underbrace{0,\ldots,0}_{9},\underbrace{1,\ldots.1}_{9},\underbrace{-1,\ldots,-1}_{9}\right\} \tag{3}$$

with $\epsilon = \frac{\eta\langle\phi^0\rangle}{\sqrt{6}M_G}$. Putting this assigned VEV in equation (2), the boundary condition of the gauge couplings α_{3C}, α_{3L} and α_{3R} corresponding to $SU(3)_C$, $SU(3)_L$ and $SU(3)_R$ respectively, are modified to:

$$\alpha_{3C}(M_U) = (1+\epsilon)\alpha_{3L}(M_U) = (1-\epsilon)\alpha_{3R}(M_U) = \alpha_G \tag{4}$$

α_G being the GUT coupling constant. We note that, the GUT coupling is modified by the gravitational correction parameterϵ. In the next section we show that, low M_I can also be achieved through this correction by fine tuning ϵ.

Additional Light particles: A scalar particle $(1,1,8) \subset 650_H$ and a fermion $(1,1,8) \subset 78_F$, (a non-standard fermion) are introduced at M_I. An electroweak triplet $(1,3,0) \subset (1,8,1) \subset 78_F$ is also taken at a lower scale M_1 between M_I and M_Z.

Using "Extended Survival Hypothesis", we now write down the particle contents of the model, given as:

Table-1: Particle contents in different mass range.

Group	Range of masses	Higgs content	Fermions content
$\mathbb{G}_{321}$	$M_Z - M_1$	$\left(1,2,\pm\frac{1}{2}\right)_{27\oplus\overline{27}}$	MSSMPs
$\mathbb{G}_{321}$	$M_1 - M_I$	$\left(1,2,\pm\frac{1}{2}\right)_{27\oplus\overline{27}}$	MSSMPs $+(1,3,0)_{78}$
$\mathbb{G}_{333}$	$M_I - M_U$	$\{(1,\overline{3},3)\oplus(1,3,\overline{3})\}_{27\oplus\overline{27}}$ $(1,1,8)_{650}$	27 $\oplus\{(1,8,1)$ $\oplus(1,1,8)\}_{78}$

In the table above MSSMP denotes Minimal Supersymmetric Standard Model particles. In the next section we discuss the evolution equation for the gauge couplings with respect to the mass scales $M_U, M_I, M_1$and M_Z as mentioned before.

3. Evolution equations of gauge couplings and model prediction

Now with the above mentioned particle content of the model, we refer to the general one-loop Renormalisation group equations (RNGE) for inverse coupling constants [7]

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{b_i}{2\pi}\ln\left(\frac{M}{\mu}\right) \tag{5}$$

within the mass μ and M (M can be of any scale $> \mu$). Here $\alpha_i = \frac{g_i^2}{4\pi}$, where g_i being the coupling constant for the ith gauge group. b_i is the one-loop beta coefficients, which takes up different values in SUSY and non-SUSY models. The general formula for $b_i\ is$ given as:

$$\begin{aligned} b_i &= -3C_2(G_i) + T(R_i)d(R_j), \quad \text{for supersymmetric case} \\ b_i &= -\frac{11}{3}C_2(G_i) + \frac{2}{3}T(F_i)d(F_j) + \frac{1}{3}T(S_i)d(S_j), \quad \text{for non-SUSY models,} \end{aligned} \tag{6}$$

where $C_2(G_i)$ is the quadratic Casimir operator for gauge groups in their adjoint representation The factors $T(F_i)$ and $T(S_i)$ are the Dynkin indices of the irreducible representation for F_i (fermion) and S_i (scalar) with respect to the symmetry G_i, respectively.

Now referring to particles in Table-1, we calculate the beta coefficients, given as:

Table-2: One-loop beta coefficients in different mass range.

Range of masses	One-loop beta coefficients
$M_Z - M_1$	$\begin{pmatrix} b_{3C} = -3 \\ b_{2L} = 1 \\ b_Y = \frac{33}{5} \end{pmatrix}$
$M_1 - M_I$	$\begin{pmatrix} b_{3C}^1 = -3 \\ b_{2L}^1 = 3 \\ b_Y^1 = \frac{33}{5} \end{pmatrix}$
$M_I - M_U$	$\begin{pmatrix} b_{3C}^U = 0 \\ b_{3L}^U = 6 \\ b_{3R}^U = 9 \end{pmatrix}$

Using the modified boundary conditions in equation (4) arising due to gravitational correction and the numerical values of one-loop beta coefficients, the evolution equations for the inverse couplings $\alpha_{3C}^{-1}(M_Z)$, $\alpha_{2L}^{-1}(M_Z)$ and $\alpha_Y^{-1}(M_Z)$ corresponding to $SU(3)$, $SU(2)$ and $U(1)$ respectively, are obtained as:

$$\alpha_{3C}^{-1}(M_Z) = \alpha_G^{-1} - \frac{3}{2\pi}\ln\left(\frac{M_I}{M_Z}\right) \tag{7a}$$

$$\alpha_{2L}^{-1}(M_Z) = (1+\epsilon)\alpha_G^{-1} + \frac{1}{2\pi}\ln\left(\frac{M_1}{M_Z}\right) + \frac{3}{2\pi}\ln\left(\frac{M_I}{M_1}\right) + \frac{6}{2\pi}\ln\left(\frac{M_U}{M_I}\right) \tag{7b}$$

$$\alpha_Y^{-1}(M_Z) = \left(1 - \frac{3}{5}\epsilon\right)\alpha_G^{-1} + \frac{33}{10\pi}\ln\left(\frac{M_I}{M_Z}\right) + \frac{42}{10\pi}\ln\left(\frac{M_U}{M_I}\right) \tag{7c}$$

In the next section, using the above evolution equations we obtain the expressions for M_U, α_G^{-1} and $\sin^2\theta_W$ in terms of the free mass parameters M_I, M_1 and the gravitational correction ϵ. For determination of the unification mass M_U and the inverse GUT coupling α_G^{-1}, we follow the standard procedure by using the key relations between the Standard Model gauge couplings at the electroweak mass scale M_Z, given as,

$$\alpha_{\text{em}}^{-1}(M_Z)\left(\sin^2\theta_W - \frac{3}{8}\right) = \frac{5}{8}\left(\alpha_{2L}^{-1}(M_Z) - \alpha_Y^{-1}(M_Z)\right)$$

$$8\left(\alpha_S^{-1}(M_Z) - \frac{3}{8}\alpha_{\text{em}}^{-1}(M_Z)\right) = 8\alpha_{3C}^{-1}(M_Z) - 3\alpha_{2L}^{-1}(M_Z) - 5\alpha_Y^{-1}(M_Z) \tag{8}$$

Here $\alpha_{em}(M_Z)$ is the electromagnetic coupling constant = 0.007816, $\alpha_S(M_Z)$ is the strong coupling constant = 0.1181, M_Z is the Z-boson mass = 91.1876 GeV and $\sin^2\theta_W$ is the electroweak mixing angle, which is predicted to be 0.23129. Now we put the expressions of the inverse SM gauge couplings $\alpha_{3C}^{-1}(M_Z)$, $\alpha_{2L}^{-1}(M_Z)$ and $\alpha_Y^{-1}(M_Z)$ from equations (7) in the above expression and simplify to obtain:

$$\ln\left(\frac{M_U}{M_Z}\right) = \frac{4\pi}{15}\left(\frac{3}{8}\alpha_{\mathrm{em}}^{-1}(M_Z) - \alpha_S^{-1}(M_Z)\right) + \frac{1}{10}\ln\left(\frac{M_1}{M_Z}\right) - \frac{1}{10}\ln\left(\frac{M_I}{M_Z}\right) \tag{9}$$

$$\alpha_G^{-1} = \alpha_S^{-1}(M_Z) + \frac{3}{2\pi}\ln\left(\frac{M_I}{M_Z}\right) \tag{10}$$

$$\sin^2\theta_W = \frac{3}{10} + \frac{1}{5}\frac{\alpha_S^{-1}(M_Z)}{\alpha_{\mathrm{em}}^{-1}(M_Z)} + \frac{\alpha_S^{-1}(M_Z)}{\alpha_{\mathrm{em}}^{-1}(M_Z)}\epsilon - \frac{7\alpha_{\mathrm{em}}(M_Z)}{10\pi}\ln\left(\frac{M_1}{M_Z}\right) - \frac{\alpha_{\mathrm{em}}(M_Z)}{10\pi}(3 - 15\epsilon)\ln\left(\frac{M_I}{M_Z}\right) \tag{11}$$

The above expressions clearly show that, the unification mass M_U and the inverse GUT coupling α_G^{-1} are independent of the gravitational correction, but depend on the free mass parameters M_1 and M_I. However the electroweak mixing angle does depend on ϵ, the gravitational correction. We fine tune ϵ and the free mass parameters $M_1$and M_I, such that $\sin^2\theta_W$ is in agreement with the accepted value 0.23129. For phenomenological viability purpose, we fix M_1 to be Tev scale i.e. 10^3 GeV and M_I as a free choice within the range $10^4 – 10^7$. We now focus to calculate M_U, α_G^{-1} such that the unification mass M_U satisfies the proton decay constraint. For this purpose, we confine ourselves to the contribution from gauge dimension-6 operator for calculating the Proton lifetime ($p \to e^+\pi^0$) due to superheavy gauge boson exchange, such that the order of the Proton lifetime τ_p by using dimensional analysis is given by

$$\tau_p = C\frac{M_U^4}{m_p^5\alpha_G^2} \tag{12}$$

Here the constant $C \sim O(1)$ which contains all the information about the flavour structure of this theory and m_p is the mass of the proton.

Using the values of M_1, M_I and ϵ in equations (9) to (11), we calculate M_U, α_G^{-1}, $\sin^2\theta_W$. The order of τ_p is also calculated from equation (12), to know the viability of the model.

Table-3: Numerical result for M_U, α_G^{-1}, $\sin^2\theta_W$ and τ_p

M_1 (GeV)	M_I (GeV)	ϵ	M_U (GeV)	α_G^{-1}	$\sin^2\theta_W$	τ_P (in years)
10^3	10^4	0 -0.861040 $\mathbf{-0.887257}$	$10^{16.24}$ $10^{16.24}$ $10^{16.24}$	10.7103 10.7103 10.7103	0.305559 0.233485 $\mathbf{0.23129}$	3.00×10^{35}
	10^5	0 -0.76113 $\mathbf{-0.78604}$	$10^{16.14}$ $10^{16.14}$ $10^{16.14}$	11.8097 11.8097 11.8097	0.303841 0.233589 $\mathbf{0.23129}$	1.45×10^{35}
	10^6	0 -0.689970 $\mathbf{-0.702064}$	$10^{16.04}$ $10^{16.04}$ $10^{16.04}$	12.9091 12.9091 12.9091	0.302122 0.23251 $\mathbf{0.23129}$	6.91×10^{34}
	10^7	0 -0.619230 $\mathbf{-0.631268}$	$10^{15.94}$ $10^{15.94}$ $10^{15.94}$	14.0085 14.0085 14.0085	0.300404 0.232608 $\mathbf{0.23129}$	3.24×10^{34}

In the table above, the value of ϵ in bold gives the experimentally allowed $\sin^2\theta_W$. Thus a specific non-vanishing value of gravitational correction parameter ensures physically admissible unification with a low M_I in presence of a TeV scale electroweak triplet fermion. The model is also compatible with proton decay time within the range 10^{34} to 10^{35} years, such that it can be accessible in Hyper-Kamiokande. experiments.

4. Conclusion

To summarise, we have proposed a SUSY E_6 GUT model with D-parity violating intermediate symmetry. It is nice to observe that inclusion of dimension-5 operator at the unification mass scale and with additional light particles, it is indeed possible to achieve low intermediate scale within the range $10^4 – 10^7$ GeV, simultaneously ensuring admissible unification. The model accommodates a TeV scale electro-weak triplet fermion, which may have phenomenological implication in collider experiments. With the start of Hyper-Kamiokande experiments, the present SUSY E_6 GUT model seems to be relevant in predicting the dimension-6 proton decay within the reach of future observation.

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