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Color Octet Scalar in E_6 Grand Unified Theory

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Abstract: We propose a minimal E_6 GUT with intermediate trinification $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ symmetry. We assume that in presence of non-renormalizable dimension-5 operator, the discrete left-right symmetry, called D-parity is spontaneously broken at the unification mass scale M_U ensuring unequal coupling between g_{3L} and g_{3R} corresponding to $SU(3)_L$ and $SU(3)_R$ respectively. It is shown that in presence of a massive weak singlet color-octet scalar, the predicted value of M_U is compatible with the accessible limit of the proton lifetime. The color octet scalar with mass of the order of few hundred GeV may suppress the production of the Higgs boson through gluon-fusion at LHC.

Keywords: D-parity, Color octet, Trinification symmetry, Gravitational correction.

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1. Introduction

Many BSM (Beyond the Standard Model) theories, involving various exotic particles have been proposed by the particle physicists in recent years in order to comply with some of the unsolved issues of the Standard model (SM). However, it is a challenging task to identify them in the Hadron colliders. Tevatron and the LHC [1] allow the production of new particles carrying QCD (Quantum Chrome Dynamics) color at a high rate. With reference to the existing theoretical constructs, there are models that have not been carefully confronted with the LHC data, involve extension of the scalar sector of the SM with electroweak singlet or doublet color octets [2]. Color octet particles are present in various extensions of the Standard Model, ranging from supersymmetric models to composite models for the electroweak sector. The simplest of those is the weak-singlet scalars, which is present in various theories including Technicolor, universal extra dimensions [3], vector-like confinement [4], weakly-interacting metastable pion models [5], or certain supersymmetric models (sgluons) [6]). These scalars can be produced in pairs through its QCD couplings to gluons, and may decay through higher dimensional operators into a pair of heavy quarks.

Considering the significance of these color octets, in the present study we investigate the existence of such color octets in a minimal E_6 grand unified model [7] with interemediate Trinification $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ symmetry [8]. The viability of these particles is investigated with respect to admissible gauge coupling unification consistent with accessible proton decay constraint. The novel feature of the model shows that in presence of the color octet scalars, it allows low intermediate scale, so as to have observable phenomenology

The paper is structured into four sections including the introduction. The next section focuses on the model framework. Section-3 deals with the renormalisation group analysis for the model to obtain the expressions of the unification mass, inverse GUT coupling constant and electroweak mixing angle, along with the numerical estimations. The last section is devoted to a discussion on the phenomenological implications of the model.

2. Model framework

We discuss here the key features of the present E_6 GUT in it's minimal form with D-parity violating intermediate trinification symmetry $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ ($\mathbb{G}_{333}$), with asymmetric left-right coupling. The model utilises the Higgs $\{650_H \oplus 27_H\}$ for symmetry breaking purpose. At the unification mass scale, in addition to the conventional Lagrangian, we introduce non-renormalizable dimension-5 operator [9],

$$\mathbb{L}_{NRO} = -\frac{\eta}{4M_G}\mathrm{Tr}\left(F_{\mu\nu}\Phi_{650}F^{\mu\nu}\right) \tag{1}$$

which induces gravitational corrections. In fact the presence of these operators show the remnants of Planck scale physics, which in turn imply the unification of the GUT model with gravity. These terms may arise as a result of compactification of some higher dimensional gauge theory. Here the scale $M_G \leq M_{Pl}$, the Planck scale $\simeq 10^{19}$ GeV, η is a dimensionless parameter, $F_{\mu\nu}$ is the field strength tensor and $\Phi(1,1,1)$ being the Higgs scalar contained in $650_H \subset E_6$. The discrete left-right symmetry, called the D-parity is spontaneously broken at the unification mass scale M_U by assigning a GUT scale VEV to D-parity odd singlet scalar $\Phi(1,1,1)$ leading to $g_{3L} \neq g_{3R}$. The next stage of symmetry breaking i.e., from $\mathbb{G}_{333} \rightarrow \mathbb{G}_{321}$ (the standard model) at the intermediate scale M_I is done by putting a non-zero VEV for the $\mathbb{G}_{321}$ neutral component of trinification multiplet $(1,\bar{3},3)$ of 27_H. It has been shown in some earlier works [10-13], that the model with minimal Higgs $(1,\bar{3},3) \subset 27_H$ could not admit phenomenologically viable gauge unification (which comes out to be the scale beyond Planck energy). In a previous work by us [14] with D-parity conserving intermediate trinification symmetry, it is shown that the problem can be solved by introducing additional scalar $(1,8,8) \subset 650_H$. However, in the present case with D-parity violating intermediate symmetry, this also does not work and we need additional particle like color octet scalar, to be introduced at a scale M_1 between the intermediate scale M_I and the weak scale M_Z. The last stage of symmetry breaking i.e., SM to low energy theory ($\mathbb{G}_{31}$) is done by assigning a non-zero VEV to the conventional SM Higgs doublet contained in $27_H \subset E_6$. The corresponding breaking pattern is given as,

$$E_6 \xrightarrow{M_U} (\mathbb{G}_{333})\, SU(3)_C \otimes SU(3)_L \otimes SU(3)_R\ (g_{3L} \neq g_{3R})$$

$$\xrightarrow{M_I} (\mathbb{G}_{321})\, SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$\xrightarrow{M_1} (\mathbb{G}_{321})\, SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$\xrightarrow{M_Z} (\mathbb{G}_{31})\, SU(3)_C \otimes U(1)_Q \tag{2}$$

As has been mentioned before, at M_U we assign VeV to the odd singlet scalar $\Phi_{650}\ (1,1,1)$, given as,

$$\langle \Phi_{650} \rangle = \frac{\langle \phi^0 \rangle}{\sqrt{6}} \mathrm{diag} \left\{ \underbrace{0,\ldots,0}_{9}, \underbrace{1,\ldots,1}_{9}, \underbrace{-1,\ldots,-1}_{9} \right\} \tag{3}$$

Putting this assigned VEV in equation (1), the boundary conditions of the gauge couplings α_{3C}, α_{3L} and α_{3R} corresponding to $SU(3)_C$, $SU(3)_L$ and $SU(3)_R$ respectively, are modified to:

$$\alpha_{3C}(M_U) = (1+\epsilon)\alpha_{3L}(M_U) = (1-\epsilon)\alpha_{3R}(M_U) = \alpha_G \tag{4}$$

where α_G is the GUT coupling constant and $\epsilon = \frac{\eta\langle \phi^0 \rangle}{\sqrt{6} M_G}$. Now using this modified boundary conditions for the gauge couplings, we can express the renormalization group equations at different mass ranges M_Z to M_1, M_1 to M_I and M_I to M_U in the next section, so as to calculate the mass scales involved in the model.

3. The renormalization group analysis and numerical prediction

In order to obtain the gauge coupling evolution we use the standard renormalization group equations (RGEs) [15] for different range of mass scales corresponding to the channel in eqn. (2). We know that in general the one-loop RGEs for inverse coupling constant valid from μ to the intermediate scale M (M can be of any scale $> \mu$ where new theory appears) as,

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{b_i}{2\pi} \ln\left(\frac{M}{\mu}\right) \tag{5}$$

Here, $\alpha_i = \frac{g_i^2}{4\pi}$, where g_i being the coupling constant for the ith gauge group. $b_i$is the one-loop beta coefficients in the mass range $\mu - M$. In the present case, within the mass scale M_I to M_U, the $\mathbb{L}_{NRO}$ will be operative in the RGE through the parameter ϵ, such that we have another ϵ dependant term in the RGE. Taking into account the above fact, after simplification, the evolution equations for the inverse couplings of the standard model i.e., α_{3C}^{-1} , α_{2L}^{-1} and α_Y^{-1} at the weak scale M_Z, are given as,

$$\alpha_{3C}^{-1}(M_Z) = \alpha_G^{-1} + \frac{b_{3C}}{2\pi} \ln\left(\frac{M_1}{M_Z}\right)$$

$$+ \frac{b_{3C}^1}{2\pi} \ln\left(\frac{M_I}{M_1}\right) + \frac{b_{3C}^U}{2\pi} \ln\left(\frac{M_U}{M_I}\right) \tag{6}$$

$$\alpha_{2L}^{-1}(M_Z) = (1+\epsilon)\alpha_G^{-1} + \frac{b_{2L}}{2\pi} \ln\left(\frac{M_1}{M_Z}\right)$$

$$+ \frac{b_{2L}^1}{2\pi} \ln\left(\frac{M_I}{M_1}\right) + \frac{b_{3L}^U}{2\pi} \ln\left(\frac{M_U}{M_I}\right) \tag{7}$$

$$\alpha_Y^{-1}(M_Z) = \left(1 - \frac{3}{5}\epsilon\right)\alpha_G^{-1} + \frac{b_Y}{2\pi} \ln\left(\frac{M_1}{M_Z}\right)$$

$$+\frac{b_Y^1}{2\pi}\ln\left(\frac{M_I}{M_1}\right)+\left(\frac{\frac{1}{5}b_{3L}^U+\frac{4}{5}b_{3R}^U}{2\pi}\right)\ln\left(\frac{M_U}{M_I}\right) \quad (8)$$

where b_i and b_i^1 with $i = 3C, 2L, Y$ are the one-loop beta coefficients between the mass range M_Z to M_1 and M_1 to M_I respectively. Similarly, b_i^U with $i = 3C, 3L, 3R$ are the one-loop beta coefficients between the mass range M_I to M_U. Using the general known formula [16] one can have the numerical values of the beta coefficients, which is given in the following table. As per the extended survival hypothesis, the detail particle content in the specified mass range, along with the beta values, are given below.

Table-1: Fields and one-loop beta coefficients in different mass range.

Group	Range of masses	Higgs content	Fermions content	One-loop beta coefficients
$\mathbb{G}_{321}$	$M_Z - M_1$	$\left(1,2,-\frac{1}{2}\right)_{27}$	SMPs	$\begin{pmatrix} b_{3C} = -7 \\ b_{2L} = -\frac{19}{6} \\ b_Y = \frac{41}{10} \end{pmatrix}$
$\mathbb{G}_{321}$	$M_1 - M_I$	$\left(1,2,-\frac{1}{2}\right)_{27}$ $(8,1,0)_{650}$	SMPs	$\begin{pmatrix} b_{3C}^1 = -6 \\ b_{2L}^1 = -\frac{19}{6} \\ b_Y^1 = \frac{41}{10} \end{pmatrix}$
$\mathbb{G}_{333}$	$M_I - M_U$	$(1,\bar{3},3)_{27}$ $\{(1,8,8) \oplus (8,1,1)\}_{650}$	27	$\begin{pmatrix} b_{3C}^U = -4 \\ b_{3L}^U = \frac{7}{2} \\ b_{3R}^U = \frac{7}{2} \end{pmatrix}$

Here SMPs within the mass range $M_Z - M_I$, denote the Standard Model Particles. Similarly, in the mass range $M_I - M_U$, the fermion 27 includes,

$$27_F = L(1,\bar{3},3) \oplus Q_L(3,3,1) \oplus Q_L^C(\bar{3},1,\bar{3})$$

Now following the standard procedure i.e. using the standard key relations, $\alpha_{\text{em}}^{-1}(M_Z) - \frac{8}{3}\alpha_{3C}^{-1}(M_Z)$, $\alpha_{\text{em}}^{-1}(M_Z) - \frac{8}{3}\alpha_{2L}^{-1}(M_Z)$ and $\alpha_{\text{em}}^{-1}(M_Z) = \frac{5}{3}\alpha_Y^{-1}(M_Z) + \alpha_{2L}^{-1}(M_Z)$, we can obtain the expressions for the unification mass scale M_U, the inverse GUT coupling constant α_G^{-1} and electroweak mixing angle $\sin^2\theta_W$. Putting the values of the beta coefficients obtained in Table-1, we have

$$\ln\left(\frac{M_U}{M_Z}\right) = \frac{4\pi}{15}\left(\frac{3}{8}\alpha_{\text{em}}^{-1}(M_Z) - \alpha_S^{-1}(M_Z)\right) - \frac{2}{15}\ln\left(\frac{M_1}{M_Z}\right) + \frac{1}{60}\ln\left(\frac{M_I}{M_Z}\right) \tag{9}$$

$$\alpha_G^{-1} = \frac{7}{15}\alpha_S^{-1}(M_Z) + \frac{1}{5}\alpha_{\text{em}}^{-1}(M_Z) + \frac{7}{30\pi}\ln\left(\frac{M_1}{M_Z}\right) + \frac{31}{30\pi}\ln\left(\frac{M_I}{M_Z}\right) \tag{10}$$

$$\sin^2\theta_W = \frac{3}{8} + \left(\frac{1}{5} + \frac{7}{15}\frac{\alpha_S^{-1}(M_Z)}{\alpha_{\text{em}}^{-1}(M_Z)}\right)\epsilon + \frac{\alpha_{\text{em}}(M_Z)}{30\pi}7\epsilon\ln\left(\frac{M_1}{M_Z}\right)$$

$$-\frac{\alpha_{\text{em}}(M_Z)}{240\pi}(545 - 248\epsilon)\ln\left(\frac{M_I}{M_Z}\right) \tag{11}$$

It is observed that for a given choice of intermediate scales M_I and M_1 (mass of the color octet), unification mass scale M_U and the corresponding inverse GUT coupling constant $\alpha_G^{-1}$are unaffected by the gravitational correction parameter ϵ. However, $\sin^2\theta_W$ value is controlled by this parameter. Thus for viable phenomenology, we fine tune ϵ and the free mass parameters $M_1$and M_I, such that $\sin^2\theta_W$ is in agreement with the accepted value 0.23129. Hence in our numerical estimation, we strictly follow this constraint in order to obtain the unification mass M_U, the inverse GUT coupling constant α_G^{-1} and the electroweak mixing angle $\sin^2\theta_W$. Further to show the viability of the model we calculate the order of the Proton lifetime τ_p by using dimensional analysis, where

$$\tau_p = C\frac{M_U^4}{m_p^5\alpha_G^2} \tag{12}$$

Here the constant $C \sim O(1)$ which contains all the information about the flavour structure of this theory and m_p is the mass of the proton. Here we confine ourselves to the contribution from gauge dimension-6 operator [17] for calculating the Proton lifetime ($p \rightarrow e^{+}\pi^{0}$) due to superheavy gauge boson exchange.

Using the experimental values of $\alpha_{\rm em}^{-1}(M_Z)$, $\alpha_S^{-1}(M_Z)$, the GUT scale M_U, the inverse GUT coupling constant α_G^{-1} and electroweak mixing angle $\sin^2\theta_W$ and τ_p are calculated with input values for M_1 (mass of the color octet $(8, 1, 0)$ scalar, M_I (the intermediate mass scale) and ϵ (gravitational correction).

Table-2: Numerically estimated value for M_U, α_G^{-1} and $\sin^2\theta_W$.

M_1 (GeV)	M_I (GeV)	ϵ	M_U (GeV)	α_G^{-1}	$\sin^2\theta_W$	τ_P (in years)
10^3	10^4	-0.125463 -0.398560 $\mathbf{-0.479531}$	$10^{16.23}$ $10^{16.23}$ $10^{16.23}$	31.2644 31.2644 31.2644	0.317806 0.251075 $\mathbf{0.23129}$	2.33×10^{36}
	10^6	-0.084233 -0.220136 $\mathbf{-0.355821}$	$10^{16.27}$ $10^{16.27}$ $10^{16.27}$	32.7791 32.7791 32.7791	0.300867 0.266051 $\mathbf{0.23129}$	3.70×10^{36}
10^5	10^6	-0.129876 -0.322114 $\mathbf{-0.352147}$	$10^{16.00}$ $10^{16.00}$ $10^{16.00}$	33.1212 33.1212 33.1212	0.288827 0.239064 $\mathbf{0.23129}$	3.15×10^{35}
	10^9	-0.032196 -0.163015 $\mathbf{-0.188465}$	$10^{16.05}$ $10^{16.05}$ $10^{16.05}$	35.3933 35.3933 35.3933	0.274517 0.23833 $\mathbf{0.23129}$	5.70×10^{35}

The value of ϵ in bold face gives the experimentally allowed $\sin^2\theta_W$. For a fixed M_I, the values of M_U, α_G^{-1} remain same even with the variations of gravitational correction ϵ, as is obvious form the equations (9) and (10). However, $\sin^2\theta_W$ varies with ϵ and for a particular value of ϵ, we get experimentally allowed electroweak mixing angle $\sin^2\theta_W$.

We can see that the model can accommodate a color octet scalar, with possible mass M_1 ranging from $10^3 - 10^5$ GeV, in tune with $M_I = 10^4 - 10^9$ GeV. The proton decay lifetime is found within a range $10^{35} - 10^{36}$ years, so as to be experimentally accessible in future.

4. Conclusion

We have explored the possibility of a massive color octet scalar in an E_6 GUT model with the effect of gravitational corrections through non-renormalizable operator. The analysis is confined to D-parity violating trinification symmetry at the intermediate scale. It is nice to mention that it is indeed possible to accommodate the color octet and simultaneously to achieve low $M_I \leq 10^9$ GeV with successful gauge unification in comply with the verifiable proton lifetime. As far as phenomenological implication is concerned, these colored scalars, would be produced in pairs with a large rate at the LHC, which may affect the Higgs boson production cross section in gluon fusion. Further the model is consistent with low intermediate scale, so that we may have testable phenomenology. It is nice to note that, the model is free from any topological defects thus ensuring cosmological stability.

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