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A Comparative Study of Mouth Organ and Beating Bell (Ghanta) from the Perspective of Cantilever

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Abstract. Neither a mouth organ nor a ghanta (beating bell) appear to be connected even remotely to a mechanical device like either a cantilever or a beam. Earlier we have shown that by treating each plate associated with a particular note in a mouth organ as a cantilever, the frequency bears a ratio of two small integers with the experimental value of the corresponding note. And here, this approach is applied to the common musical instrument Ghanta whose frequency is derived by treating it as a bent beam, i.e., two cantilevers joined together. This has application in selecting different frequencies of the octave while building musical instruments in which circular plates are used or construction of circular roofs. It can be matched with a frequency monitor and from this and various parameters the Young's modulus can also be measured and this can be a promising experiment. Another important limitation is that the formula may not be valid for large bending as Hooke's law is no longer valid.

I. Introduction

The elastic behaviour of materials plays an important role in everyday life. All engineering designs require precise knowledge of the elastic behaviour of building materials. For example, while designing a building the structural details of the coulumns, beams and supports require knowledge of the materials used [1]. The condition for using simple bending theory is that the beam is subject to pure bending, i.e., shear force is zero and there is no torsional or axial load. Again, the material is to be isotropic and homogeneous. It should obey Hooke's law (It is linearly elastic and will not deform plastically) and cross sections of beam remain plain during bending [2].

If a solid returns exactly to its original shape when the external force is removed, it is said to be perfectly elastic. However no solid is perfectly elastic. It behaves so only when the deformation produced is small. To get a feel for the orders of magnitude involved, consider steel rod of length $1\,m$ and diameter1 cm.If you hang a medium size car(mass$\sim 3000kg$)from the end of such rod, the rod will stretched ,but only 0.05% [1].

II. Methodology

Now we are going to derive the formula for the frequency of a popular instrument, i.e., beating bell (ghanta) by modelling the 'bent beam' as a beating bell. Here our aim is to explore the musical property of a 'Ghanta' (in Odia) or Beating bell and therefore to determine relationship between its various physical parameters like mass of the plane surface, diameter l, Young's modulus of the material of which it is made up of (generally copper) and thickness of the plane surface d. This is done by considering the ideal case of a beam loaded at the middle point.Here the middle point of the beam is loaded with a weight W.The reaction at each knife edge is $\frac{W}{2}$ kg-wt acting vertically upward [3].

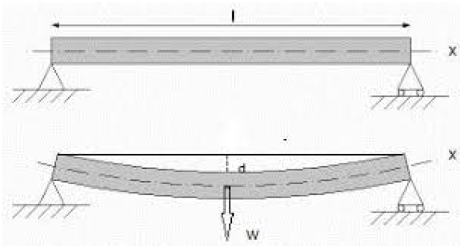

Fig. 1: Bending of beam

Let l be the distance between the two knife edges $K_1$and K_2. Taking a section at C, at a distance x from D let us now consider the equilibrium at the part CB. Since the beam is fixed at D, the load $\frac{W}{2}$ exerts a torque on CB tending to rotate it anticlockwise.

The magnitude of restoring torque is $\frac{YI}{\rho}$, where Y is the Young's modulus of the material of the beam, I is the geometrical moment of inertia of the section C [4].

Now,

$$\frac{W}{2}\left(\frac{l}{2}-x\right)=\frac{YI}{\rho}$$

But $\rho = \frac{1}{\frac{d^2y}{dx^2}}$ is the radius of curvature.

Then we get

$$\frac{d^2y}{dx^2} = \frac{W}{2YI}\left(\frac{l}{2} - x\right)$$

This finally gives

$$y=\frac{W}{2YI}\left(\frac{lx^2}{4} - \frac{x}{6}\right)$$

At B we have $x = \frac{l}{2}$, and the elevation is maximum. Let it be equal to δ.

Then $$\delta = \frac{Wl^3}{4Ybd^3} \quad (1)$$

Now from equation (1) we get

$$W = \frac{\delta bd^3}{l^3} \times 4Y$$

But we are considering the beating bell and hence let us take a small segment $dr = rd\theta = b$

Where we are considering $l = 2r$

Then we get for the small segment a small weight

$$dW = \frac{\delta 4Yd^3}{l^3} rd\theta = \delta \frac{4Yd^3}{8r^3} rd\theta$$

Or, $$dW = \delta \frac{Yd^3}{2r^2} d\theta$$

Now considering the generalisation for a real beating bell we integrate

$$\int dW = \int_0^{2\pi} \delta \frac{Yd^3}{2r^2} d\theta$$

$$\Rightarrow W = \delta \frac{Yd^3}{2r^2} \int_0^{2\pi} d\theta = \delta \frac{Yd^3}{2r^2} \times 2\pi$$

$$\Rightarrow W = \left(\frac{\pi Yd^3}{r^2}\right)\delta \quad (2)$$

Till now we are considering the beating bell of very light mass and we obtain equation (2). Here W is the weight which is nothing but the force [5].So, we can say that equation (2) is nothing but the Hooke's law ,

$$F = -kx$$

Where the constant $k = \left(\frac{\pi Yd^3}{r^2}\right)$

Now actually in real case the flat surface of the bell is very heavy. Letthe massof the flat surface be M_b. As the symmetry of the beam leads to the symmetry of the 'beating bell', the mass M_b is located at the centre of the flat portion. So the extra weight added to the equation (2) leads to

$$W + M_b g = -(\frac{\pi Y d^3}{r^2})\delta \quad (3)$$

Suppose we beat the bell once and release it to oscillate, then it will move with its natural frequency. Here $W = 0$. So equation (3) becomes

$$M_b g = -(\frac{\pi Y d^3}{r^2})\delta \quad (4)$$

Then the time period can be calculated by putting the value of k in the general formula

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{M_b \times r^2}{\pi Y d^3}}$$

From which one can calculate the frequency to be

$$\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\pi Y d^3}{M_b \times r^2}} \quad (5)$$

From equation (5) it is seen that for a beating bell

$$\nu \propto \sqrt{Y}$$

$$\nu \propto d^{\frac{3}{2}}$$

$$\nu \propto \frac{1}{\sqrt{M_b}}$$

And $\nu \propto \frac{1}{r}$

A musical scale is a sequence of frequencies which have a particularly pleasing effect on human ear. A widely used musical scale called a diatonic scale has eight frequencies covering an octave. Each frequency is called a note. The table below shows seven frequencies where the eighth has double of the first note Sa.

The octave in sound is given as follows [6].

Table 1. The Octave

Symbol	Indian name	Frequency in Hertz
C	Sa(ସା)	256
D	Re(ରେ)	288
E	Ga(ଗା)	320

F	Ma(ମା)	$341\frac{1}{3}$
G	Pa(ପା)	384
A	Dha (ଧା)	$426\frac{2}{3}$
B	Ni(ନି)	480

So, by choosing suitable material ($Y, r,$ M_b and d) one can calculate the frequency of beating bell in commensurate with the octave table [7].

III. Results and Discussion

The two dissimilar appearing acoustic instruments (Mouth Organ and Beating Bell) have a common origin from a physical instrument called cantilever which is generally used for measurement of elastic constants like Young's Modulus. But here its acoustical property is explored.

The cantilever and bent beam has many similarities. In Cantilever if we put a weight W at the fixed end then in doing the calculation for its free end depression, we have to put the whole weight on the formula [8]. But in case of a bent beam as it has weight on its middle and has two supports at its ends we have to divide the weight equally in portions which can act like two cantilevers.

Now a property of the cantilever or a beam (which is nothing but two cantilevers joined together) is its elastic constant k. Here in case of a cantilever after doing all the calculations [3], we have found the depression

$$\delta = \frac{4Wl^3}{Ybd^3}$$

This can be written in the form of Hooke's Law as

$$W = -\frac{Ybd^3}{4l^3}\delta$$

This remains in the form $F_1 = -k_1\delta$

To get an expression for bent beam we have to put $\frac{W}{2}$ in place of w and $\frac{l}{2}$ in place of l in the above equation and this gives

$$W = \frac{4Ybd^3}{l^3}$$

This also remains in the form

$$F_2 = -k_2\delta$$

IV. Conclusion

The formula that we derived for the 'Ghanta's frequency from the perspective of bent beam is useful in many ways. It may be useful to select different frequencies of the octave while building musical instrument in which circular plates are used [9]. For example, this frequency of beating bell can be used to derive the natural frequencies of circular roofs' construction. It can be matched with a frequency monitor and from this and various parameters the Young's modulus can be measured and this can be a promising experiment. Another important limitation is that the formula may not be valid for large bending as Hooke's law is no longer valid [10,11].

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