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Physics of Chaos: From Non-Physicists to Physicists

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"Chaos was the law of nature : Order was the dream of man"
Henry Adams

Abstract. We live in a dynamic world, often described as "unpredictable" and "chaotic". The word "Chaos" come to mind is confusion, disorder, and lack of control. It is the common phenomenon in non-linear science, and special motion of non-linear systems. From human perspective point of view, we do not visualize the greater framework of the system within its boundaries. Chaos is an intrinsically richness related to its structure with their wide range of potential behaviours. In view of the above, the paper discusses various aspects: (a) the basics of chaos (physics) and presents mainly the importance of the nonlinearities nature in the physical systems. Finally, we will discuss for an example of Lorenz Equations (LE) chaotic system. We use a MATLAB computer program to simulate the behaviour of the LE and observe how it is sensitive to initial conditions with graphical aspects (phase planes and trajectory profiles) as a case study.

Keywords: Chaos, system, simple pendulum, state spaces and trajectories.

1. Introduction

It is important to understand certain preliminary scientific concepts in order to obtain a better understanding of physical systems and chaotic dynamical systems in particular. To interpret any basic scientific research, a sound background of the theory of knowledge is essential. One of the principles of the theory of nowledge that is important for physical modeling concerns the relationship between scientific theories and reality. The most of the basic laws of nature are *deterministic*, which allows determining exactly what will happen next from knowledge of present conditions. One of the axioms of the modern science is description of a physical system and the possibility of a deeper understanding of the system and the prediction of the evolutions. In general, if to obtain complete solutions of the equations governing the mechanics of a system with many degrees of freedom, then intractable problems are faced. A system with three degrees of freedom subjected to chaotic behaviour, in which a small perturbation in the initial conditions grows exponentially, so that the detailed long-term behaviour is essentially unpredictable.

Everyone knows what „chaos‟ is. The prominent meanings that come to mind are confusion, disorder, and lack of control. It is a natural phenomenon that provides the very interesting property of sensitivity to initial conditions (SIC). As the behaviour is sensitive to the initial conditions, any disturbance however small will grow exponentially, and leads to a different trajectory over time.

It is the common phenomenon in non-linear science. The best-known example of chaos is the Butterfly Effect. Chaos was first conceptualized and defined through mythology, which described the origins (or birth) of humankind. As per Alvin Toffler, We might characterize today’s breakdown of industrial or “Second Wave” society as civilization “bifurcation, and the rise of a more differentiated, “Third Wave” society as a leap to new “dissipative structures” on world scale. And, if we accept this analogy, might we look upon the leap from Newtonianism to Prigoginianism in the same way? Mere analogy, no doubt. But illuminating, nevertheless. Chaos offers deep insights into these questions-insights that bear on the nature of each as creative beings. For a human being, creativity is about getting beyond what we know, getting to the “truth” of things. That’s where chaos comes in. Being a theoretical physics concept, let us focus on as: *Twentieth-century theoretical physics came out of the relativistic revolution and the quantum mechanical revolution. It was all about simplicity and continuity (in spite of quantum jumps). Its principal tool was calculus. Its final expression was field theory. Twenty-first-century theoretical physics is coming out of the chaos revolution. It will be about complexity and its principal tool will be the computer. Its final expression remains to be found. Thermodynamics, as a vital part of theoretical physics, will partake in the transformation.*

A dynamical system is a system which evolves with time from a prescribed initial condition(s) with a well defined rule(s). It is important to understand certain preliminary scientific concepts in order to obtain a better understanding of physical systems with its chaotic dynamical systems.

Why have scientists, engineers, physicists and mathematicians become intrigued by chaos? The answer to that question has two parts: (a) The study of chaos has provided new conceptual and theoretical tools enabling us to categorize and understand complex behaviour; (b) chaotic behaviour seems to be universal. Even chaotic behaviour shows qualitative and qualitative universal features, which are independent of the details of the particular system.

2. The Discovery of Chaos

According to the Encyclopaedia Britannica the word "chaos" is derived from the Greek "*χαοσ*" and originally meant the infinite empty space which existed before all things. The later Roman conception interpreted chaos as the original crude shapeless mass into which the Architect of the world introduces order and harmony. In modern usage which we will adopt chaos denotes a state of disorder and irregularity.

The laws of science aim at relating cause and effect, which predict various events thousands of years in advance. But there are other natural phenomena that are not predictable though they obey the same laws of physics like the weather, the flow of a mountain stream, the roll of a dice phenomenon. It was believed that precise predictability can in principle be achieved, by gathering and processing sufficient amount of information.

For theoretical physicists the revolution started a few decades ago. Chaos is a purely mathematical concept; it is an undeniable mathematical fact. For a continuous dynamical system, the necessary number of degree of freedom for observing chaos is three or more. Chaos is a few decades old. It belongs in the field of theoretical physics, where it stands for undeniable, theoretical yet mathematical facts.

Literally, the word ‟chaos‟ means the total disorder or utter confusion. The unpredictable and complex evolution of deterministic systems is commonly referred to as chaos. The randomness associates with a chaotic system come from the intrinsic dynamics of the system. The fundamental characteristic of a chaotic system is its extreme sensitivity to the initial conditions, i.e., the phase space trajectories started with slightly different initial conditions will diverge exponentially. Thus a very small variation in the initial condition produces an infinitely large effect on the long term behaviour of the system. For the same reason, the long term prediction of a chaotic system is practically impossible.

The first numerical evidence of chaos theory was given by Edward Lorenz, the father of chaos theory, a meteorologist at MIT in the early 60‟s. In the computer's memory, six decimal places were stored whereas only three appeared on the printout. An error of one part in a thousand had changed his weather patterns drastically. Lorenz called his discovery "the butterfly effect" - the notion that a butterfly flapping its wings in Bombay will set off a tornado in Japan a week later. Technically, the butterfly effect is called sensitive dependence on initial conditions, which is one of the hallmarks of chaos.

He described chaos in these terms: "when the present determines the future, but the approximate present does not approximately determine the future". He observed certain non repeating solutions while simulating a truncated version of atmospheric convection.

However, the history of chaos theory starts from the time of the renowned French mathematician Henry Poincare. There was a belief that the complete evolution of a physical system can be predicted if its dynamical equations and the corresponding initial conditions are given. The predictability of the dynamical system definitely depends on the evolution of this error in computations. After Lorenz" discovery of chaos in the convection model, chaos has been observed in many nonlinear systems such as lasers, population models and electronic circuits and so on.

The advent of chaos introduces us to a new type of attractor – a *strange attractor* or a chaotic attractor. Geometrically a strange attractor is *a fractal*, i.e. it reveals more detail as it is increasingly magnified. In strange attractors, arbitrarily nearby orbits diverge exponentially fast and so stay together for only a short time.

2.1 Definition of Chaos

There is still no universally accepted definition for chaos. The definition given is more in terms of the descriptive properties of chaotic nonlinear systems. A general definition of chaos is "Chaos is defined as the quality of a deterministic mathematical system in which an extreme sensitivity to initial conditions exists". The mathematical calculations involved in modelling chaos theory requires large numbers of calculations, often iterated (repeated), thus this field has blossomed in parallel with the computer revolution. Let us define chaos as follows: Chaos: A dynamical system S is chaotic if: (a) The set of periodic points in S is dense. (b) S is transitive. (c) S depends sensitively on initial conditions. (d) The linear equation obeys the rule:

$$f(a_1x_1 + a_2x_2) = a_1 f(x_1) + a_2 f(x_2), a_1, a_2 \text{ are constants}$$

2.2 Mathematical View of Chaos

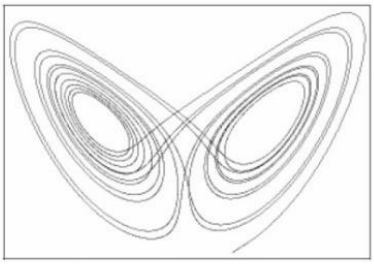
As Chaos is so young that mathematicians still have not decided on the definition of chaos!! Based on the classical notions, the evolution of an initial condition x_0 under an operator Φ' does **not** behave chaotically if:

$\Phi'(x_0)$ goes to an equilibrium when $t \to \infty$

$\Phi'(x_0)$ goes to a periodic orbit when $t \to \infty$

$\Phi'(x_0)$ escapes to $t \to \infty$

A system is chaotic if (a subset of the) orbits are confined to a bounded region, but still behave unpredictably. A good example is the Lorenz Equations with σ = 10, *r* = 28 and *b* = 8/3. The displays for these parameters in Lorenz equation are shown in the below figures. Arbitrary orbits seem to accumulate on an object called the butterfly attractor, Figure 1. Chaotic behaviour means that this is not true, in the sense that fairly quickly the two orbits start to behave very differently and there is no trace or "memory" of the fact that they were once so close. This property is called sensitive dependence on initial conditions and can be roughly translated as "all decimal places matter."



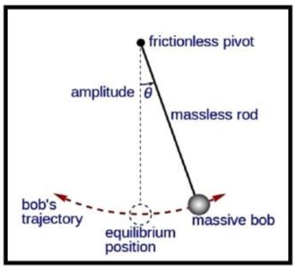
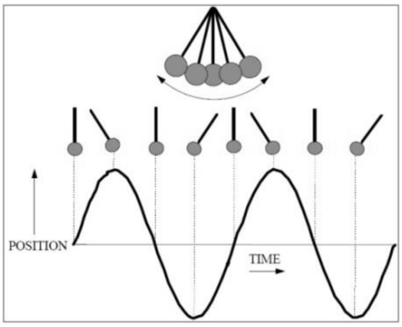


Fig. 1: The Lorenz Attractor Figure 2: The Ideal Pendulum

2.3 Phase Plane Analysis of Ideal Pendulum

The system of an ideal pendulum, simply known as simple pendulum, Figure 2, is a weight-a bob of mass-suspended from a pivot so that it can swing freely. The bob moves in two dimensions, so the system "we need only two pieces of information to completely describe the physical state of the system: *position* and *velocity*." These two observable quantities are often referred to as phase variables. As only one of the coordinates is independent, so the system has only one degree of freedom.



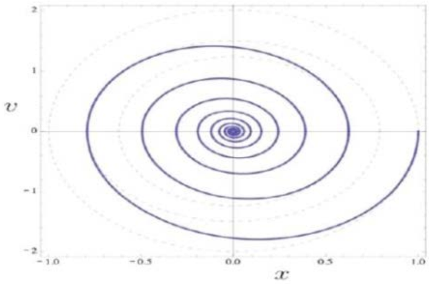


Fig. 3: Oscillation of Ideal Pendulum **Fig. 4:** Phase-Plane of Ideal Pendulum

The phase-space is simply a way of graphically representing the way a system behaves. Let us think a simple frictionless pendulum. This is a dynamical system; in other words a system that changes with time. One way to describe this system is to graph how the bob position changes from moment to moment, Figure 3. If we define the position of the pendulum as zero when the bob is hanging straight down, then left of centre is negative, and right of centre is positive. As time progresses, the bob swings to one side then the other. Graphically this looks like a sine wave, as shown, Figure 3. Most people are familiar with the sine wave as one form of *periodic* behaviour.

For a pendulum steadily losing energy to friction, all trajectories spiral inward to a point that represents a steady state, Figure 4. We call this point an *attractor* of the system and it is a single point; the attractors exist in the phase space, and one of the most powerful inventions of modern science.

In phase-space, its trajectory would not close, but would spiral inward until the bob reached a steady state of zero position and zero velocity shown in Figure 4. One can think of the attractor of a periodic system as that point that the system would eventually collapse to in phase-space if no external forces act upon it.

2.4 Modelling of Lorenz Equations

In 1963, the physicist E. Lorenz revolutionized the situation demonstrating that the qualitative nature of atmospheric turbulence which obeys the Navier-Stokes complex partial differential equations is representable by a simple nonlinear model of the third order (Lorenz equation):

$$\frac{dx}{dt} = \sigma(y - x); \frac{dy}{dt} = -xz + rz - y; \frac{dz}{dt} = xy - bz \qquad (1)$$

where σ, r and b are parameters. We choose σ = 10, b = 8/3 and r = 28 and let the initial conditions be $x(0) = y(0) = z(0) = 0.1$ at t = 0. The solutions of system (1) look like nonregular oscillations. The trajectories in the state (phase) space can approach the limit set (attractor) featuring very sophisticated form. Time step is h = 0.02 and total number of steps are nn = 2000. Using MATLAB, we create the following script chaos.m.

2.5 Performance Analysis: Phase Space and Trajectories Profiles

These hidden orderly patterns in chaotic behaviour can be presented in the so-called phase space. Phase spaces are abstract mathematical spaces, that is a set of structured points, normally with a high number of coordinates (each particular variable taken into account by the model is associated to a different coordinate), so that each point in this abstract space represents a complete and detailed state which the analysed system could eventually reach.

At the basic level, phase-space is simply a way of graphically representing the way a system behaves. Take for example a simple, frictionless pendulum. This is a dynamical system; in other words a system that changes with time. One way to describe this system is to graph how the bob position changes from moment to moment.

MATLAB is a simulation tool which can be used to simulate linear or non-linear, continuous dynamic systems. The MATLAB codes for the proposed Lorenz equations are established based on Equation (1). This model has three parameters which will affect the behaviour in addition to the parameters associated with the original equations. Simulating continuous chaotic systems requires that an appropriate step size be chosen. This is a hidden parameter within the system model (typically set to 0.01s or 0.02s), used to obtain the desired behaviour.

2.6 MATLAB Source Codes for Lorenz Equations Simulation (chaos.m)

```
% MATLAB Codes for Simulation of Lorenz Equations
% Aim: to solve the Lorenz equations
% to plot phase planes and trajectory profiles
% dx/dt=sigma*(y-x); dy/dt=-x*z+r*x-y; dz/dt=x*y-b*z
sig=10.0;   %  Parameter
b=8/3;      %   -do-
r=28;       %   -do-
t(1)=0.0;   %  Initial t
x(1)=0.1; y(1)=0.1; z(1)=0.1; % Initial x,y,z
dt=0.02;  % Time step
nn=2000; % Number of time steps
for k=1:nn % Time loop
    fx=sig*(y(k)-x(k));                % RHS of x equation
    fy=-x(k)*z(k)+r*x(k)-y(k);  % RHS of y equation
    fz=x(k)*y(k)-b*z(k);           % RHS of z equation
    x(k+1)=x(k)+dt*fx;             % Find new x
    y(k+1)=y(k)+dt*fy;             % Find new y
    z(k+1)=z(k)+dt*fz;             % Find new z
    t(k+1)=t(k)+dt;                  % Find new t
end                                       % Close time loop
  % Phase Planes Profiles
  %
  figure(1)
  plot(x,y,'-k') % Plot x vs y
  xlabel('\bf x'); ylabel('\bf y'); % Label axes
  grid on
  title('\bf Phase Plane Simulation of Lorenz
Equations(x~y)') % Title
  %
  figure(2)
  plot(y,z,'-k') % Plot y vs z
  xlabel('\bf y'); ylabel('\bf z'); % Label axes
  grid on
  title('\bf Phase Plane Simulation of Lorenz
Equations(y~z)') % Title
  %
```

```
figure(3)
  plot(z,x,'-k') % Plot z vs x
  xlabel('\bf z'); ylabel('\bf x'); % Label axes
  grid on
  title('\bf Phase Plane Simulation of Lorenz Equations(z~x)')
% Title
  %
  figure(4)
  plot3(x,y,z,'-k')
  grid on
  title('\bf Phase Plane Simulation of Lorenz
Equations(x~y~z)') % Title
  xlabel('\bf x'); ylabel('\bf y'); zlabel('\bf z');
  %
  % Trajectories Profiles
  %
  figure(5)
  plot(t,x,'-k') % Plot x vs t
  grid on
  title('\bf Trajectory Simulation of Lorenz Equations(t~x)') %
Title
  xlabel('\bf t'); ylabel('\bf x'); % Label axes
  %
  figure(6)
  plot(t,y,'-k') % Plot y vs t
  grid on
  title('\bf Trajectory Trajectory of Lorenz Equations(t~y)') %
Title
  xlabel('\bf t'); ylabel('\bf y'); % Label axes
  %
  figure(7)
  plot(t,z,'-k') % Plot z vs t
  grid on
  title('\bf Trajectory Simulation of Lorenz Equations(t~z)') %
Title
  xlabel('\bf t'); ylabel('\bf z'); % Label axes
  %
   % End of the Program
```

2.7 Simulation Outputs

The phase planes for the Lorenz equations shown in Figures 5-8 and the trajectories of various variables with time shown in Figures 9-11 generated by MATLAB codes. These shapes are the same as those for the corresponding continuous systems and which confirms the chaotic behaviour of the discrete systems is stable.

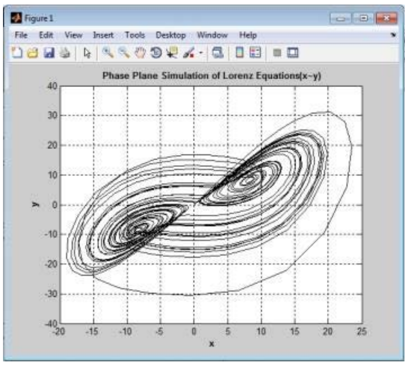


Fig. 5: Phase Plane of x~y

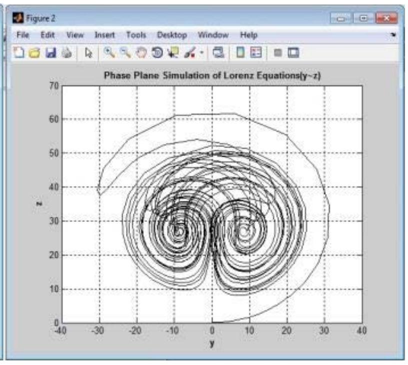


Fig. 6: Phase Plane of y~z

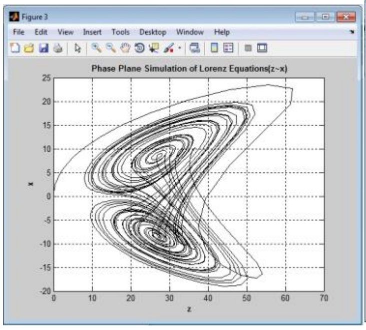


Fig. 7: Phase Plane of z~x

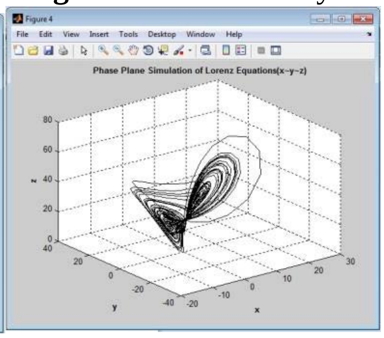


Fig. 8: Phase Plane of x~y~z

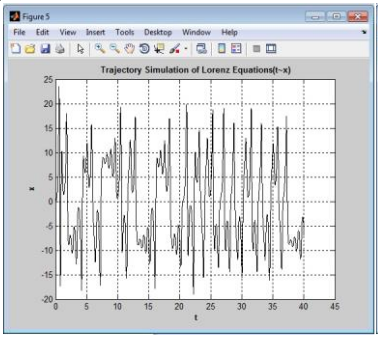


Fig. 9: Trajectory Profile of t~x

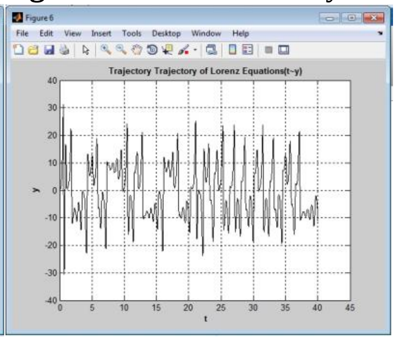


Fig. 10: Trajectory Profile of t~y

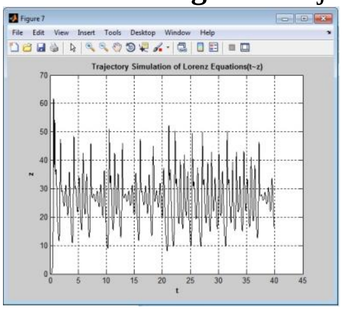


Fig.11: Trajectory Profile of t~z

3.0 Applications of Chaos

Chaos has a body of knowledge comprised of theoretical elements including systems like nonlinearity, diversity, disorder, disequilibrium, instability, and unpredictability. It is applied in many scientific disciplines like geology, mathematics, programming, microbiology, biology, computer science, economics, engineering, finance, meteorology, philosophy, physics, politics, population dynamics, psychology, and robotics. For chaos to be theoretically accepted with a body of knowledge requires a model which demonstrates

applications.

4.0 Conclusion

The discovery of chaos has far reaching implications in many branches of science and engineering. It has provided physicists, mathematicians and scientists with a new way of studying and understanding the natural world. The discovery of chaos has created a new paradigm in scientific thinking. The world is not as strictly deterministic as was once believed under classical mechanics. Computers have played a major role in the discovery and subsequent developments in this field. The computer is to chaos what cloud chambers and particle accelerators are to particle-physics. It has been implicated in areas ranging from heart failure, meteorology, economic modelling, and population biology to chemical reactions, neural networks, fluid turbulence and more speculatively even manic-depressive

behaviour. Chaos has provided us with a new way of looking at nature, which has helped us to find order in places where we earlier found only disorder in many different areas ranging from sciences, mathematics and engineering to social systems.

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