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Study of the Mixing Parameters of $D^0 - \overline{D}^0$ Oscillation in Dirac formalism

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Abstract. In this paper we investigate the mixing parameters of $D^0 - \overline{D}^0$ oscillation in Dirac formalism. In this context for the present study, we assume that the constituent quark –anti quark inside a meson is independent confined by an average potential of the form

$$V_q(r) = V_s(r) + \gamma^0 V_V(r)$$

Where $V_s(r)$ and $V_V(r)$ are scalar and vector components of the potential respectively. Since the present model considers the confining potential to be an equal admixture of scalars and vector components i.e. $V_s(r) = V_V(r) = \frac{1}{2}(a^2 r + V_0)$. We discuss here the mass oscillation of the neutral open charm meson and the integrated oscillation rate using spectroscopic parameters deduced from our earlier study. If CP symmetry is violated, the oscillation rates for meson produced as D^0 and $\overline{D}^0$ can differ, further enriching the phenomenology. The mixing parameters of $D^0 - \overline{D}^0$ oscillation, x_q, y_q and R_M are in very good agreement with BaBar and Belle collaboration results.

Keywords: Mesons, oscillation rates, mixing parameters of $\boldsymbol{D^0 - \overline{D}^0}$ oscillation.

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1. Introduction

Very recently, experiments at LHCb[1] have reported large number of D_J resonances in the mass range of 2.0 GeV/c^2 to 4.0 0 GeV/c^2 of which many of them belong to natural excited states of D meson while quite a number of them belong to unnatural states [1].The recent experimental data on excited D- states are partially inclusive and require more detailed analysis involving their decay properties. The understanding of the weak transition from factor of heavy mesons is important for a proper extraction of the quark mixing parameters, for the analysis of non-leptonic decays and CP violatingeffects.QCD sum rule [2] is non perturbative approach to evaluate hadron properties by using the correlator of the quark currents over the physical vacuum and it is implemented with the operator product expansion(OPE). Lattice QCD [3] is also non-perturbative approach to use a discrete set of space time points (lattice) to reduce the analytically intractable path integrals of the continuum theory to a very difficult numerical computation.QCD sum rules are suitable for describing the low q^2 region of the form factors; lattice QCD gives good predictions for high q^2. As a result these methods do not provide for a full picture of the form factors and more significant, for the relations between the various decay channels. Potentials models provide such relations and give the form factors in the full q^2 range.

A different D^0 decay channel [4] has been reported by three experimental groups as evidence of $D^0 - \overline{D}^0$ oscillation. We discuss here the mass oscillation of the neutral open charm meson and the integrated oscillation rate using our spectroscopic parameters deduced from the present study. In the standard model, the transitions$D^0 - \overline{D}^0$ and $\overline{D}^0 - D^0$ occur through the weak interaction. The neutral D meson mix with their antiparticle leading to oscillations between the mass eigenstates [5]. In the following, we adopt the notation introduced in [5], and assume CPT conservation in our calculations. If CP symmetry is violated, the oscillation rates for meson produced as D^0 and $\overline{D}^0$ can differ, further enriching the phenomenology. The study of CP violation in D^0 oscillation may lead to an improved understanding of possible dynamics beyond the standard model [6].

The work is organized in this paper as follows: In section-II we first of all provide the basic formalism which deals with a brief outline of potential model giving the zeroth order quark dynamics and we describe mixing parameters of $D^0 - \overline{D}^0$ oscillations. Finally section –III embodies our results and conclusion.

2. Theoretical Framework

To first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the meson through the quark Lagrangian density

$$\mathcal{L}_q^0(x) = \overline{\Psi}_q \left[\frac{i}{2}\, \gamma^\mu\, \vec{\partial}_\mu \; - V(r) - m_q\right] \Psi_q(x) \qquad (1)$$

In this context for the present study, we assume that the constituent quark – antiquark inside a meson is independently confined by an admixture of scalar and vector parts in linear potential [7]

$$V(r) = \frac{1}{2}(1+\gamma_0)\,(a^2 r + V_0), \quad a > 0 \tag{2}$$

In the stationary case, the spatial part of the quark wave functions $\Psi(\vec{r})$ satisfies the Dirac equation given by

$$\left[\gamma^0 E_q - \vec{\gamma}\,.\,\vec{P} - m_q - V(r)\right]\Psi_q(\vec{r}) = 0 \tag{3}$$

It can be transformed into a convenient dimensionless form given as [1]

$$\frac{d^2\, g(\rho)}{d\rho^2} + \left[\epsilon - \rho - \frac{k(k+1)}{r^2}\right] g(\rho) = 0 \tag{4}$$

and

$$\frac{d^2\, f(\rho)}{d\rho^2} + \left[\epsilon - \rho - \frac{k(k-1)}{r^2}\right] f(\rho) = 0 \tag{5}$$

Where $\rho = r/r_0$ is a dimensionless variable with the arbitrary scale factor chosen conveniently as

$$r_{0q} = \left(\lambda_q a^2\right)^{-1/3} \tag{6}$$

and ϵ is a corresponding dimensionless Eigen value defined by as

$$\epsilon = \left(E_D - m_q - V_O\right)\left(\lambda_q/a^4\right)^{\frac{1}{3}} \tag{7}$$

The corresponding mass of the quark-antiquark system can be written as

$$M_{Q\bar{q}} = E_D^Q + E_D^{\bar{q}} \tag{8}$$

We have adopted the same parametric form of the confined gluon propagators which are given by [2,3]

$$D_0(r) = \left(\frac{\alpha_1}{r} + \alpha_2\right)\, exp(-r^2 c_0^2/2) \tag{9}$$

and

$$D_1(r) = \frac{\gamma}{r}\, exp(-r^2 c_1^2/2) \tag{10}$$

With $\alpha_1 = 0.036$, $\alpha_2 = 0.056$, $c_0 = 0.1017$ GeV, $c_1 = 0.1522$ GeV, $\gamma = 0.0139$ GeV [1].

2.1 Mixing Parameters of $D^0 - \overline{D}^0$ Oscillation

The time evolution of the neutral D −meson doublet is described by a Schrödinger equation with an effective 2× 2 Hamiltonian given by [8].

$$i\frac{d}{dt}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} \tag{11}$$

where M and Γ matrices are Hermitian, and are defined as

$$\left(M - \frac{i}{2}\Gamma\right) = \left[\begin{pmatrix} M_{11}^q & M_{12}^{q*} \\ M_{12}^q & M_{11}^q \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11}^q & \Gamma_{12}^{q*} \\ \Gamma_{12}^q & \Gamma_{11}^q \end{pmatrix}\right] \tag{12}$$

CPT invariance imposes

$$M_{11} = M_{22} \equiv M, \Gamma_{11} = \Gamma_{22} \equiv \Gamma \tag{13}$$

The off-diagonal elements of these matrices describe the dispersive and absorptive parts of $D^0 - \overline{D}^0$ mixing [9]. The two eigenstates D_1 and D_2 of the effective Hamiltonian matrix are $\left(M - \frac{i}{2}\Gamma\right)$ given by

$$|D_J\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}(p|D^0\rangle + q|\overline{D}^0\rangle), \tag{14}$$

$$|D_2\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}(p|D^0\rangle - q|\overline{D}^0\rangle), \tag{15}$$

The corresponding eigen values are

$$\lambda_{D_1} = m_1 - \frac{i}{2}\Gamma_1 = \left(M - \frac{i}{2}\Gamma\right) + \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \tag{16}$$

$$\lambda_{D_2} = m_2 - \frac{i}{2}\Gamma_2 = \left(M - \frac{i}{2}\Gamma\right) - \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \tag{17}$$

Where $m_1(m_2)$ and $\Gamma_1(\Gamma_2)$ are the mass and width of $D_1(D_2)$, respectively, and

$$\frac{q}{p} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right)^{\frac{1}{2}} \tag{18}$$

From equations (16) and (17), one can get the differences in the mass and width which are given as

$$\Delta m \equiv m_2 - m_1 = -2Re\left[\frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\right] \tag{19}$$

$$\Delta\Gamma \equiv \Gamma_2 - \Gamma_1 = -2\,Im\left[\frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\right] \tag{20}$$

The calculation of the dispersive and absorptive parts of the box diagrams yields the following expressions for the off-diagonal element of the mass and decay matrices; for example if $s/\bar{s}$ as the intermediate quark state then [10],

$$M_{12} = -\frac{G_F^2\, m_W^2\, \eta_{D^0}\, m_{D^0}\, B_{D^0}\, f_{D^0}^2}{12\pi^2}\, S_0(m_s^2/m_W^2)\, (V_{us}^* V_{cs})^2 \tag{21}$$

$$\Gamma_{12} = \frac{G_F^2\, m_W^2\, \eta'_{D^0}\, m_{D^0}\, B_{D^0}\, f_{D^0}^2}{8\pi} [(V_{us}^* V_{cs})^2] \tag{22}$$

where G_F is the Fermi constant, m_W is the Wboson mass, m_c is the mass of c quark, m_{D^0}, f_{D^0} and $B_{D^0}$are the D^0 mass, weak decay constant and bag parameter respectively.The known function $S_0(x_q)$ can be approximated very well by $0.784x_q^{0.76}$ [11] and V_{ij} are the elements of the CKM matrix [12]. The parameter η_{D^0} and η'_{D^0} correspond to the glunoic corrections. The only non-negligible contributions to M_{12} are from box diagrams involving$s(\bar{s}), d(\bar{d}), b(\bar{b})$ intermediate quarks in Fig. (1) .The phases of $M_{12}$and Γ_{12} satisfies

$$\phi_M - \phi_\Gamma = \pi + O\left(\frac{m_c^2}{m_b^2}\right) \tag{23}$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the $K^0 - \bar{K}^0$ system, the heavy state is expected to have a smaller decay width than that of the light state: Hence, $\Delta\Gamma = \Gamma_1 - \Gamma_2$ is expected to be positive in the Standard Model. Further, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2}\, \frac{m_c^2}{m_W^2}\, \frac{1}{S_0(m_s^2/m_W^2)} \sim O\left(\frac{m_q^2}{m_t^2}\right) \tag{24}$$

Is small ,and a power expansion of $|q/p|^2$ yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\phi_M - \phi_\Gamma) + O\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right) \tag{25}$$

Therefore, considering both Eqs.(23) and (24), the $CP -$violating parameter given by

$$1 - \left|\frac{q}{p}\right|^2 \simeq Im\left(\frac{\Gamma_{12}}{M_{12}}\right) \tag{26}$$

Is expected to be very small: $\sim O(10^{-3})$ for the $D^0 - \bar{D}^0$ system. In the approximation of negligible CP violation in mixing, the ratio $\Delta\Gamma/\Delta m$ is equal to the small quantity $|\Gamma_{12}/M_{12}|$ of Eqs. (24); it is hence independent of CKM matrix elements, i.e., the same for the $D^0 - \bar{D}^0$ system.Theoretically, the hadron lifetime (τ_{D^0}) is related to $\Gamma_{11}(\tau_{D^0} = 1/\Gamma_{11})$, while the observable Δm and $\Delta\Gamma$are related to $M_{12}$and Γ_{12} as [13]

$$\Delta m = 2|M_{12}| \tag{27}$$

and $$\Delta\Gamma = 2|\Gamma_{12}| \tag{28}$$

The gluonic correction can find from by different model like Wilson coefficient and evolution of Wilson coefficient from the new physics scale [14]. We have used values of gluonic correction ($\eta_{D^0} = 0.86; \eta'_{D^0} = 0.21$) from [15,16]. The bag parameter $B_{D^0} = 1.34$ is taken from the lattice result of [17], while the pseudoscalar mass (M_{D^0}) and the pseudoscalar decay constant (f_D) of D mesons are the values obtained from our present study using relativistic independent quark model using Martin like potential. The values of $m_s(0.1$ GeV), $M_W(80.403 GeV)$ and the CKM matrix elements $V_{cs}(1.006)$ and V_{us} (0.2252) are taken from the Particle Data Group [13]. The resulting mass oscillation parameters Δm are tabulated in Table-2 with latest experimental results.

The integrated oscillation rate $\left(\chi_q\right)$ is the probability to observe a $\bar{D}$ meson in a jet initiated by a $\bar{c}$ quark. As the mass difference Δm_D is a measure of the frequency of the change from a D^0 into a $\bar{D}^0$ or vice versa. This change is reflected in either the time-dependent oscillations or in the time-integrated rates corresponding to the di-lepton events having the same sign. The time evolution of the neutral states from the pure $\left|D^0_{phys}\right\rangle$ or $\left|\bar{D}^0_{phys}\right\rangle$ state at t = 0 is given by

$$\left|D^0_{phys}(t)\right\rangle = g_+(t)|D^0\rangle + \frac{q}{p} g_-(t)|\bar{D}^0\rangle \tag{29}$$

$$\left|\bar{D}^0_{phys}(t)\right\rangle = g_+(t)|\bar{D}^0\rangle + \frac{p}{q} g_-(t)|D^0\rangle \tag{30}$$

Which means that the flavor states remain unchanged (g_+) or oscillate into each other (g_-) with time-dependent probabilities proportional to

$$g_+(t) = e^{\frac{-\Gamma t}{2}} e^{itM_{D^0}} \cos(t\Delta m/2) \tag{31}$$

$$g_-(t) = e^{\frac{-\Gamma t}{2}} e^{itM_{D^0}} \sin(t\Delta m/2) \tag{32}$$

Starting at t = 0 with initially pure D^0, the probability for finding a $D^0(\bar{D}^0)$ at time $t \neq 0$ is given by $|g_+(t)|^2$ $|g_-(t)|^2$ .Taking $\left|\frac{q}{p}\right| = 1$, one gets

$$|g_+(t)|^2 = \frac{1}{2} e^{\frac{-\Gamma t}{2}} [1 \pm \cos(t\Delta m)] \tag{33}$$

Conversely, from an initially pure $\overline{D}^0$ at t=0, the probability for finding a $\overline{D}^0(D^0)$ at time $t \neq 0$ is also given by$|g_+(t)|^2 (|g_-(t)|^2)$.The oscillation of D^0 or $\overline{D}^0$ shown by Eqs. (33) give Δm directly. Integrating $\left|g_\pm(t)\right|^2$ from t=0 and t=∞, we get

$$\int_0^\infty \left|g_\pm(t)\right|^2 dt = \frac{1}{2}\left[\frac{1}{\Gamma} \pm \frac{\Gamma}{\Gamma^2+(\Delta m)^2}\right] \tag{34}$$

Where $\Gamma = \Gamma_D = (\Gamma_1 + \Gamma_2)/2$. The ratio

$$r_0 = \frac{D^0 \leftrightarrow \overline{D}^0}{D^0 \leftrightarrow D^0} = \frac{\int_0^\infty |g_-(t)|^2 dt}{\int_0^\infty |g_+(t)|^2 dt} \frac{x^2}{2+x^2} \tag{35}$$

$$x_q = x = \frac{\Delta m}{\Gamma} = \Delta m \tau_D \quad , \quad y_q = \frac{\Delta\Gamma}{2\Gamma} = \frac{\Delta\Gamma \tau_D}{2} \; \chi_q = \frac{x_q^2+y_q^2}{2(x_q^2+1)} \tag{36}$$

Reflects the change of pure D^0 into $\overline{D}^0$, or vice versa.

The time-integrated mixing rate relative to the time-integrated right-sign decay rate for semileptonic decays [13] is

$$R_M = \int_0^\infty r(t)\; dt = \int_0^\infty \; |g_-(t)|^2 \left|\frac{q}{p}\right|^2 \; dt \tag{37}$$

$$R_M = \int_0^\infty \frac{e^{-t}}{4} (x^2 + y^2)\; t^2 \left|\frac{q}{p}\right|^2 \simeq \frac{1}{2}\left((x^2 + y^2)\right) \tag{38}$$

In the Standard Model, CP violation in charm mixing is small and $\left|\frac{q}{p}\right| \approx 1$

For the present estimation of these mixing parameters x_q , y_q and χ_q we employ our predicated Δm values and the experimental average lifetime of PDG [13] of the D −meson.

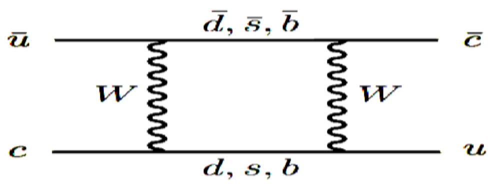

Fig.-1

Table -1: The fitted model for parameters for D-mesons

System parameters	D
Quark mass (in GeV)	$m_{u/d} = 0.2875$ and $m_c = 1.8247$
Potential strength(a)	0.241+B GeV
V_0	0.375GeV
Centrifugal parameters (B)	(n*0.153) Gev^{-1} for $l = 0$ $(n + l) * 0.1267$ Gev^{-1} for $l \neq 0$
$\sigma(j - j$ coupling strength)	0.0055 Gev^3 for l=0 0.0946 Gev^3 for $l \neq 0$

Table-2: Mixing Parameters x_q , y_q , χ_q ,R_M

	$\Delta\mathbf{M}(\mathbf{GeV})$	$\mathbf{x_q}$	$\mathbf{y_q}$	$\boldsymbol{\chi_q}$	$\mathbf{R_M}$
Present	4.521×10^{-3}	3.41×10^{-3}	4.03×10^{-3}	1.34×10^{-5}	1.34×10^{-5}
[18]		$(0.80\pm0.29)\%$	$(0.33\pm0.24)\%$		$0.804\pm0.311 \times 10^{-3}$
[19]					$0.13\pm0.22\pm0.20\times 10^{-3}$

3. Results and Conclusion

We the CP-violation parameter in $\left|\frac{q}{p}\right|$ (0.9996) in the case D^0 and $\overline{D}^0$ decay shows no evidence for CP-violation and provides the most stringent bounds on the mixing parameter x_q , y_q and mixing rate (R_M) are very good agreement with BABAR , Bellel and other collaboration as shown in the Table-2. However due to larger uncertainty in the experimental values make difficult for us to draw a continuous remark on this mixing parameters of neutral open charm meson in found to be one of the successful attempt to extract the effective quark anti-quark interaction in the case of heavy-light flavors mesons. Thus present study is an attempt to indicate the importance of spectroscopic (strong interaction) parameters in the weak decay process.

Finally we look forward to see future experimental support in favour of many of our predictions the spectral states and decay properties of the open charm mesons.

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References

[1] Y Sun, Xi Liu, and T Matsuki, *Phys. Rev*. **D 88**, 094020 (2013)

[2] P Gelhausen, A Khodjamiriana, AA Pivovarov, D Rosenthal, *Eur. Phys. J. C* **74**, 2979 (2014)

[3] Graham Moir et al., *JHEP*. **05**, 021 (2013)

[4] B Aubert et al., *Phys. Rev*. **D 80**, 071103 (2009)

[5] KA Olive et al., *Chin. Phys'*. **C 38**,090001 (2014), J Beringer et al., *Phys. Rev*. **D 86**, 010001 (2012)

[6] E Golowich, JA Hewett, S Pakvasa and AA Petrov, *Phys. Rev*. **D 76**, 095009 (2007)

[7] SN Jena, S Panda and TC Tripathy. *Nucl.Phys*. **A 658**, 249 **(**1999)

[8] G Buchalla et al., *Eur. Phys. J*. **C 57**, 309 (2008)

[9] I Bigi, and AI Sanda, *Phys. Lett*. **B 171**, 320 (1986)

[10] AJ Buras, W Slominski and H Steger, *Nucl. Phys*. **B 245**, 369 (1984)

[11] T Inami and C S Lim , *Theor. Phys* **65**, 297 (1981) [Erratum ibid. 65 (1981) 1772]

[12] M Kobayashi and K Maskawa, *Prog. Theor. Phys*. **49**, 652 (1973)

[13] KA Olive et al., *Chin. Phys.* **C 38**, 090001 (2014), J Beringer et al., *Phys. Rev*. **D 86**, 010001 (2012)

[14] E Golowich, JA Hewett, S Pakvasa and AA Petrov, *Phys. Rev*. **D 76**, 095009 (2007)

[15] M Blanka et al., *Phys. Lett*. **B 657**, 8 (2007)

[16] S Herrlich, U Nierste, *Nucl. Phys*. **B 419**, 292 (1994)

[17] AJ Buras, *Phys. Lett.* **B 566**, 115 (2003)

[18] LM Zhang et al., *Phys. Rev. Lett*. **99**, 131803 (2007)

[19] U Bitenc et al., *Phys. Rev*. **D 77**, 112003 (2008)