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Decay width of Higgs decaying into two gluons and two photons

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Abstract. The discovery of Higgs boson at the LHC has opened a new chapter in particle physics in which the properties of the Higgs boson are used to study new physical phenomena. In this paper, the extremum (maximum and minimum) values of decay width of Higgs decaying into two gluons and two photons are evaluated theoretically using spin – ½ triangle loop function and spin – 1 triangle loop function.

Keywords: Higgs decay; LHC physics; standard model; quantum field theory.

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1. Introduction

The Higgs boson is a scalar particle with spin 0 and mass given by $M_H = \sqrt{2\lambda}\,\mathrm{v}$, where λ is the Higgs self–coupling parameter. The expectation value of the Higgs field is given by $\mathrm{v} = \left(\sqrt{2}\,G_F\right)^{-1/2} \approx 246\,\mathrm{GeV}$ which is fixed by the Fermi coupling G_F and can be determined with a precision of $0.6\,ppm$ from muon decay measurements [1–3]. Since the coupling λ is a free parameter in the standard model (SM), the mass of Higgs boson is not predicted in the SM. It is interesting to observe that in the SM one needs to assume that the mass term in the potential is negative in order to trigger electroweak symmetry breaking (EWSB). In some theories beyond the SM (BSM), such as supersymmetry, the analogue of the Higgs mass parameter can be made negative dynamically. The Higgs boson couples to the fundamental particles with their masses. This is a new type of interaction which is very weak for ordinary particles, such as up and down quarks, and electrons, but strong for heavy particles such as the W and Z

bosons and the top quark. The dominant mechanisms for Higgs boson production and decay involve the coupling of H to W, Z and/or the third generation quarks and leptons. The Higgs boson coupling to gluons [4,5] is induced by a one – loop graph in which H couples to a virtual $t\bar{t}$ pair. The Higgs boson coupling to photons is also generated via loops, although in this case the one – loop graph with a virtual W^+W^- pair provides the dominant contribution [6] and the one involving a virtual $t\bar{t}$ pair is subdominant.

At the tree level, the Higgs does not couple to the gluons which constitute a very important decay mode [7], especially for low M_H. For the SM particle content, the Lagrangian is constructed by imposing the twin restrictions of gauge invariance and renormalizability.

The quantum field theory is only an effective one, operative up to scales that we can probe experimentally. Hence, higher – dimensional terms are possible as long as they are allowed by the symmetries. Similar to the $H \to gg$, the decay $H \to \gamma\gamma$ too proceeds at 1 – loop order or higher. Being an electroweak process, it is even more suppressed compared to the former. The functional behaviour is a testament to the fact that the loop functions $h_{1/2}(x)$ and $h_1(x)$ vary only for $x > 1$.

The discovery of Higgs boson marked a new era of exploration in fundamental physics. Therefore, it is necessary to study all of Higgs properties, such as the mass and individual decay widths, detailed information on the coupling magnitudes etc. [8]. The mass of Higgs boson is around 125 GeV and the spin-parity are spin zero and even parity. Recently, the total Higgs decay width has received considerable experimental and theoretical attention [8-15]. In this paper, the extremum (maximum or minimum) values of decay width of Higgs decaying into two gluons and two photons are determined theoretically.

The paper is organised as follows: In Sec. 2, maximum and minimum values of spin – ½ triangle loop function and spin -1 triangle loop function has been evaluated. In Sec. 3, we evaluate the values of decay width of Higgs decaying into two gluons by using the spin $-1/2$ triangle loop function $h_{1/2}(x)$ and its extremum values. From these values we get the extremum values of the decay width of Higgs decaying into two gluons. In Sec. 4, we evaluate the values of decay width of Higgs decaying into two photons by considering the spin – 1 triangle loop function $h_1(x)$, $h_{1/2}(x)$ and their extremum values. From these values we get the extremum values of the decay width of Higgs decaying into two photons. Finally, we present our conclusions in Sec. 5.

2. SPIN – ½ Triangle loop function and spin - 1 triangle loop function

The maximal or minimal values of the decay width of Higgs decaying into two gluons and two photons depend upon maximum and minimum values of spin – ½ triangle loop function $h_{\frac{1}{2}}(x)$ and spin –1 triangle loop function $h_1(x)$. In order to get maximal or minimal decay widths, we have to find $h_{\frac{1}{2}}(x)max$, $h_{\frac{1}{2}}(x)min$, $h_1(x)max$ and $h_1(x)min$. Using these values along with general values of $h_{\frac{1}{2}}(x)$ and $h_1(x)$ we will be able to calculate different values of decay width of Higgs decaying into two gluons and two photons. Out of these values one will be maximum and one will be minimum. The spin – ½ triangle loop function can be written as [7]

$$h_{\frac{1}{2}}(x) = 2x[1 + (1 - x)f(x)], \tag{1}$$

where

$$f(x) = \begin{cases} \left(\sin^{-1}\frac{1}{\sqrt{x}}\right)^2, & x \geq 1 \\ -\frac{1}{4}\left[log\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi\right]^2, & x < 1. \end{cases} \tag{2}$$

In equations (1) and (2) x is defined as $x_q \equiv 4m_q^2/M_H^2$ where m_q is mass of quark and M_H is mass of Higgs boson taken. The maximum and the minimum values of $h_{\frac{1}{2}}(x)$ can be calculated by using

$$h_{\frac{1}{2}}(x) = 2x[1 + (1 - x)f(x)],$$

where

$$f(x) = \left(\sin^{-1}\frac{1}{\sqrt{x}}\right)^2, \quad \text{when} \quad x = \operatorname{cosec}^2(\theta \pm \phi) \geq 1 \tag{3}$$

$$= -\frac{1}{4}\left[log\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi\right]^2, \text{ when } x = -4\exp\{-(m \pm n)\} \;\; x < 1 \tag{4}$$

.

$\theta = -\frac{\sqrt{x-1}}{2(1-2x)}$, $\phi = \frac{\sqrt{9x-5}}{2(1-2x)}$, $m = -\frac{\sqrt{1-x}}{(2x-1)}$, and $n = \frac{\sqrt{5-9x}}{(2x-1)}$. Here, θ and ϕ are angles used in equation (3) where as m and n are variable quantities used in equation (4). The parameters θ, ϕ, m, and n are function of $x = x_q \equiv 4m_q^2/M_H^2$. The values of x obtained from equations (3) and (4) has been used in equations

(1) and (2) to find the values of $h_{\frac{1}{2}}(x)$. Using (+) sign we are getting minimum value of $h_{\frac{1}{2}}(x)$ and using (–) sign we are getting maximum value of $h_{\frac{1}{2}}(x)$ from equations (3) and (4).

The spin –1 triangle loop function $h_1(x)$ is defined as [7]

$$h_1(x) = -[2 + 3x + 3x(2 - x)f(x)], \tag{5}$$

where $f(x) = \left(\sin^{-1}\frac{1}{\sqrt{x}}\right)^2$ since we are considering the case $x \geq 1$.

The maximum and the minimum values of $h_1(x)$ can be calculated by using

$$h_1(x) = -[2 + 3x + 3x(2 - x)f(x)],$$

$$f(x) = \left(\sin^{-1}\frac{1}{\sqrt{x}}\right)^2, \; x = cosec^2(\xi \mp \eta) \geq 1$$

where $\xi = \frac{(x-2)}{4(x-1)^{3/2}}$, $\eta = \frac{\sqrt{(x-2)^2+8(x-1)^2}}{4(x-1)^{3/2}}$ (6)

Here, ξ and η are angles used in equation (6). The parameters ξ and η are function of $x = x_q \equiv 4m_q^2/M_H^2$. The values of x obtained from equation (6) has been used in equations (1) and (2) to find value of $h_1(x)$. Using (–) sign we are getting maximum value of $h_1(x)$ and using (+) sign we are getting minimum value of $h_1(x)$ from equation (6). The maximum and minimum values of spin – 1/2 triangle loop function and spin – 1 triangle loop function for different particles has been evaluated and are shown in Table 1.

Table 1: Maximum and minimum values of spin – ½ triangle loop function and spin - 1 triangle loop function for different particles

Particles	**Mass (MeV)**	$\mathbf{h_{\frac{1}{2}}(x)}\,max$	$\mathbf{h_{\frac{1}{2}}(x)}\,min$
u	2.3	-1.3251	-42.31334
d	4.8	-1.3251	-42.31334
c	1.235×10^3	-1.32263	-42.267
s	95	-1.325	-42.3131
t	173.5×10^3	1.3763	1.3445
b	4.18×10^3	-1.2973	-41.7834
e^-	0.511	-1.325	-42.31334
μ^-	106	-1.325	-17.952
τ^-	1.777×10^3	-1.32	-17.9267

Particle	Mass (MeV)	$h_1(x)max$	$h_1(x)min$
$W^{\pm}$	80.39×10^3	-7.282	-7.642

3. Decay width of Higgs decaying into two gluons

The decay width of the Higgs decaying into two gluons is given by [7, 16–19]

$$\Gamma(H \rightarrow gg) = \frac{G_F \alpha_S^2 M_H^3}{36\sqrt{2}\pi^3} \left| \sum_q \frac{3}{4} h_{\frac{1}{2}}(x_q) \right|^2, \tag{7}$$

where the (coherent) sum runs over all quarks, $x_q \equiv 4m_q^2/M_H^2$ and $h_{\frac{1}{2}}(x)$ the spin – ½ triangle loop function is defined in equation (1). For $x \rightarrow 0, h_{\frac{1}{2}}(x) \rightarrow 0$. A massless quark would have a vanishing contribution. Hence, the decay width of the Higgs decaying into gluons will be zero for a massless quark and $\Gamma(H \rightarrow gg)$ is dominated by the top loop. The Higgs could decay into a $q\bar{q}$ pair, which rescatters to give two gluons and leaves a final state re scattering phase. For $x \geq 1$, the loop function $h_{\frac{1}{2}}(x)$ is a slowly varying function with $h(1) = 2$ and $h(\infty) = 4/3$. As the quark mass is increased to a large quantity, the loop does not diverge. Different values of decay width of Higgs decaying into two gluons are evaluated by using the spin – ½ triangle loop function and its extremum values (given in Table 1) in equation (7) and are shown in Table 2.

Table 2: Decay width of Higgs decaying into two gluons

Sl. No.	Expressions for decay width of Higgs decaying into two gluons	Decay width (MeV)
1.	$\Gamma(H \rightarrow gg) = \frac{G_F \alpha_S^2 M_H^3}{36\sqrt{2}\pi^3} \left| \sum_q \frac{3}{4} h_{\frac{1}{2}}(x_q) \right|^2$	16.242
2.	$\Gamma_1(H \rightarrow gg) = \frac{G_F \alpha_S^2 M_H^3}{36\sqrt{2}\pi^3} \left| \sum_q \frac{3}{4} h_{\frac{1}{2}}(x_q)max \right|^2$	222.5312
3.	$\Gamma_2(H \rightarrow gg) = \frac{G_F \alpha_S^2 M_H^3}{36\sqrt{2}\pi^3} \left| \sum_q \frac{3}{4} h_{\frac{1}{2}}(x_q)min \right|^2$	359.1×10^3

In the above table we have shown three values of decay width of Higgs decaying into two gluons for quarks having non-zero mass. In first case we have used general value of the function $h_{1/2}(x)$. In the second case we have used the maximum value of $h_{1/2}(x)$. In the third case we have used minimum value of $h_{1/2}(x)$. The minimum and maximum values of the decay width of Higgs decaying into two gluons are found to be $16.242\,\text{MeV}$ and $359.1\,\text{GeV}$ respectively.

4. Decay width of Higgs decaying into two photons

The Higgs decaying into two photons $H \to \gamma\gamma$ is similar to $H \to gg$ which proceeds at $1-$ loop order or higher. The decay width of the Higgs decaying into two photons is given by [4, 7, 20]:

$$\Gamma(\mathrm{H} \to \gamma\gamma) = \frac{\mathrm{G_F}\alpha_{\mathrm{em}}^2 \mathrm{M_H^3}}{128\sqrt{2}\pi^3}\left|\sum_{\mathrm{f}} \mathrm{N_c^{(f)}} \mathrm{Q_f^2} \mathrm{h}_{1/2}(\mathrm{x_f}) + \mathrm{h}_1(\mathrm{x_W})\right|^2, \quad (8)$$

where the (coherent) sum runs over all quarks, $h_{\frac{1}{2}}(x)$ is spin – ½ triangle loop function and is defined by equation (1), where $h_1(x)$ is spin –1 triangle loop function and is defined in equation (5).

The summation formula is applicable for all fermions, with charge Q_f and $N_c^{(f)} = 1\ (3)$ for leptons (quarks). Different values of decay width of Higgs decaying into two photons are evaluated by using the spin – ½ triangle loop function, spin –1 triangle loop function and their extremum values (given in Table 1) in eq. (8) and are shown in Table 3.

Table 3: Decay width of Higgs decaying into two photons

Sl. No.	**Expressions for decay width of Higgs decaying into two photons**	**Decay width ($\mathbf{10^{-6}}$GeV units)**
1	$\Gamma(\mathrm{H} \to \gamma\gamma) = \frac{\mathrm{G_F}\alpha_{\mathrm{em}}^2 \mathrm{M_H^3}}{128\sqrt{2}\pi^3}\left\|\sum_{\mathrm{f}} \mathrm{N_c^{(f)}} \mathrm{Q_f^2} \mathrm{h}_{1/2}(\mathrm{x_f}) + \mathrm{h}_1(\mathrm{x_W})\right\|^2$	9.171

2	$\Gamma_1(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) max + h_1(x_W) max \right\|^2$	44.28
3	$\Gamma_2(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) min + h_1(x_W) min \right\|^2$	12427.79
4	$\Gamma_3(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) max + h_1(x_W) min \right\|^2$	46.5424
5	$\Gamma_4(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) min + h_1(x_W) max \right\|^2$	12390.373
6	$\Gamma_5(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) + h_1(x_W) max \right\|^2$	6.4242
7	$\Gamma_6(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) + h_1(x_W) min \right\|^2$	7.304

8	$\Gamma_7(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) max + h_1(x_W) \right\|^2$	51.1
9	$\Gamma_8(H \to \gamma\gamma) = \frac{G_F \alpha_{em}^2 M_H^3}{128\sqrt{2}\pi^3} \left\| \sum_f N_c^{(f)} Q_f^2 h_{1/2}(x_f) min + h_1(x_W) \right\|^2$	12500.5031

In the above table we have taken different values of the functions $h_{1/2}(x)$ and $h_1(x)$ to get different values of the decay width of Higgs decaying into two photons. In first case we have taken the general values of $h_{1/2}(x)$ and $h_1(x)$, in second case we have taken maximum values of $h_{1/2}(x)$ and $h_1(x)$, in third case we have taken minimum values of $h_{1/2}(x)$ and $h_1(x)$. In fourth case we have taken maximum value of $h_{1/2}(x)$ and minimum value of $h_1(x)$ and in fifth case we have taken minimum value of $h_{1/2}(x)$ and maximum value of $h_1(x)$. In sixth case we have taken general value of $h_{1/2}(x)$ and maximum value of $h_1(x)$. In seventh case general value of $h_{1/2}(x)$ and minimum value of $h_1(x)$ has been taken. Similarly in eighth and ninth cases we have taken general values of $h_1(x)$ respectively with maximum and minimum values of $h_{1/2}(x)$. From the combination of different values of $h_{1/2}(x)$ and $h_1(x)$ we get different values of the decay width of Higgs decaying to two photons. The minimum and maximum values of the decay width of Higgs decaying into two photons are found to be 6.4242×10^{-6} GeV and $12500.5031 \times 10^{-6}$GeV respectively.

5. Conclusions

In this paper, we have evaluated the values of decay width of Higgs decaying into two gluons and two photons theoretically using spin – ½ triangle loop function, spin – 1 triangle loop function and their extremum values. From our calculations, we have found that the minimum and maximum values of decay width of Higgs decaying into two gluons are 16.242 MeV and 359.1 GeV respectively. Similarly, the minimum and maximum values of decay width of Higgs decaying into two photons are found to be 6.4242×10^{-6} GeV and $12500.5031 \times 10^{-6}$ GeV respectively. We hope in the near future, experimental searches at the LHC run II will explore these decays with high precision measurements. These facts lead to enrichment in the phenomenology of Higgs boson.

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