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Artillery Projectile Trajectory Motion: A Multi-disciplinary Computational Approach of a Dynamic Physical System

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Abstract : One of the most important methods in current scientific and technological research is process of modeling and simulation of real experiment as well as real experimental device. System approach, modeling and simulation are discipline with its own theory and research methodology. This paper focuses to the theory of the process of modeling and simulation, visualization and model validation and verification of real experiment and experimental device. Multidisciplinary approach is also discussed. Step by step there will be depicted the process of creation of static and dynamic mathematical model of the real experimental device - *Artillery Projectile Trajectory Motion (APTM)*. Mathematical model is supplemented the simulation model written in MATLAB. Visualization is a part of the simulation model. Validation of the mathematical model as well as verification of the simulation model is also discussed.

Keywords: Mathematical model, computer simulation, multidisciplinary approach, system approach, artillery projectile trajectory motion.

1. Introduction

The terms *system*, *modeling*, *simulation*, *multidisciplinary approach* are important in current approach to scientific, technological and professional practice. Many are realizing that modeling and simulation is becoming an important tool in solving and understanding numerous and diverse problems. Modeling and simulation (M&S), is becoming one of the academic programs of choice for students in all disciplines. M&S is a discipline with its own body of knowledge, theory, and research methodology. To engage modeling and simulation the mathematical model of the real system has to be first created. Models are approximations for the *real dynamic physical system*. The model is then followed by *simulation*, which allows for the repeated observation of the model. After the simulations model is *verified*, a third step takes place and that is *visualization* of the model and the real system.

The ability to define a system, to build a mathematical model and to create a simulation model develops logical thinking skills and imagination and is an inseparable part of a student's study skills for those studying the specializations "Applied Informatics". In this paper we first briefly introduce the theory of modeling and simulation as a method of multidisciplinary investigation of a real experimental device.

Secondly we introduce a case study illustrating the step-by-step process of modeling and simulations of a real experimental device - *Artillery Projectile Trajectory Motion (APTM)*. The device will be described from the real system point of view, then the dynamic mathematical model will be presented and finally the computer simulation model created by programming in MATLAB with visualization.

2. Modeling and Simulation as Multidisciplinary Approach

2.1 *Modeling*

Modeling is a method that is often used in professional and scientific practice in many fields of human activity. The main goal of modeling is describe the content, structure and behavior of the real system representing a part of the reality.

The first step in the process of computer modeling is creation of mathematical model of the studied real system. The model can be obtained either theoretically based on basic physical properties of the system, or numerically by means of the measured values. Determination of parameters of theoretical model developed from empirical data is called system identification. The mathematical model must adequately describe the dependency system outputs on its inputs. Models of physical systems are usually established as systems of mathematical equations are derived in this paper.

2.2 *Simulation*

The process of modeling is closely related to the simulation. Simulation can be understood as process of executing the model. Simulation enables representation of the modeled real system and its behavior in real time by means of computer. The simulation enables also visualization and deriving of the model.

A typical simulation model can be written both through specialized programming languages that were designed specifically for the requirements of simulations, or the simulation model can be created in standard programming languages like MATLAB. In this paper the real system is simulated in MATLAB for application. From the above considerations, it is clear that simulation is a process that runs on the computer. In some publications, therefore, can be found the term "computer simulation". It generally is valid that computer simulation is a computer-implemented method used for exploring, testing and analysis of properties of mathematical models that describe the behavior of the real systems, which cannot be solved using standard analytical tools.

The simulation models represented by executable computer program have to be isomorphic with the mathematical model that is a representation. It means that the mathematical model and simulation model have to represent the real system, its elements, internal interactions and external interaction with the environment in the same way. Simulation has from the scientific point of view several functions. We will focus in this paper two of them and they are: (a) Replacing the real experiment; (b) The development of educational process.

2.2.1 *Function of Simulation - Replacement of the Real Experiment*

This is an important and indispensable feature of simulations and simulation model because it allows realize a situation that cannot be investigated using conventional real experimental devices. The main advantage of simulations is that simulations model allows changing of input parameters, visualization and optimization the effects of the real experiment. The simulation is usually safety and cheaper.

2.2.2 *Function of Simulation - Development Educational Process*

The simulation is very useful from educational point of view. Using the simulation model and visualization of simulation results on the screen, students can better understand the basic processes and systems and develop their intuition. It is also essential that the teaching by means of simulation is much cheaper and faster than the teaching carried by real experiment. In some cases providing the real experiment cannot be feasible.

Despite the fact that experimental education in the laboratory cannot be completely replaced (because students acquire manual dexterity, they learn to work with real laboratory instruments, they learn to plan, implement and evaluate realistic experiment), the simulation is a part and basic methods of scientific knowledge. Students can easily learn theoretical foundations of the laboratory tasks. The simulation model of the real laboratory task can help them to check some of the operations performed in the laboratory. This can reduce the direct lessons in the laboratory only to necessary time for their own experimental measurements. Alternatively, only the simulation models can realize lessons. In this case, it is important to note that students are deprived of contact with the real device, so that they will not get a full picture of the implementation of the experimental measurements.

2.2.3 *Model Verification and Validation*

Verification and validation are important aspects of the process modeling and simulation. They are essential prerequisites to the credible and reliable use of a model and its results.

(i) *Verification* : In modeling and simulation, verification is typically defined as the process of determining if executable simulation model is consistent with its specification - e.g. mathematical model. Verification is also concerned with whether the model as designed will satisfy the requirements of the intended application. Verification is concerned with transformational accuracy, i.e., it takes into account simplifying assumptions executable simulation model. Typical questions to be answered during verification are: (a) Does the program code of the executable simulation model correctly implement the mathematical model? (b)Does the simulation model satisfy the intended uses of the model?(c)Does the executable model produce results when it is needed and in the required format?

(ii) *Validation* : In modeling and simulation, validation is the process of determining the degree to which the model is an accurate representation of the real system. Validation is concerned with representational accuracy, i.e., that of representing the real system in the mathematical model and the results produced by the executable simulation model. The process of validation assesses the accuracy of the models. The accuracy needed should be considered with respect to its intended uses, and differing degrees of required accuracy may be reflected in the methods used for validation. Typical questions to be answered during validation are:(a)Is the mathematical model a correct representation of the real system? (b)How close does the simulation executable model to the behavior of the real system produce the results? (c) Under what range of inputs are the model's results credible and useful?

Validation and verification are both ultimately activities that compare one thing to another. Validation compares real system and mathematical model. Verification compares mathematical model and executable simulation model. Sometimes validation and verification are done simultaneously in one process. Validation of the mathematical model as well as verification of the simulation model of our real dynamic physical system - *Artillery Projectile Trajectory Motion* - are to be done simultaneously by the comparison of the dependencies of the quantities theoretically calculated from the simulation model with the dependencies of quantities experimentally measured on projectile motion. The whole process of transformation from a real system, the simulation model and its visualization is shown in Figure 1.

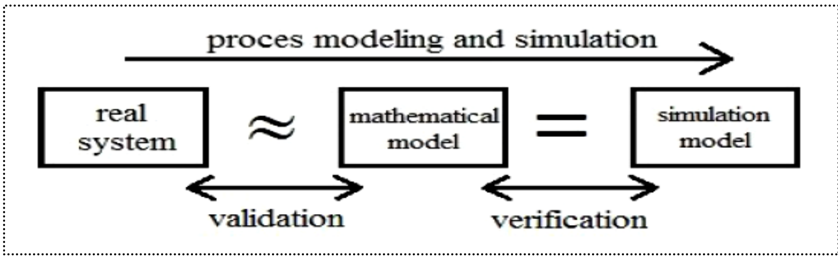

Fig. 1: Process Modeling and Simulation

It is noted that the mathematical model that reflects the real system has some limitations and simplifying assumptions (the real system and mathematical model are in homomorphic relation). In contrast, the simulation model is only the computer expression of the mathematical model (the mathematical model and simulation model are in isomorphic relationship)

2.3 *Multidisciplinary Approach*

Another important benefit associated with the process of modeling and simulation of real experiments is a multidisciplinary approach, without which the process of identifying the real system using mathematical and simulation model and cannot be realized. This is also emphasized in this paper.

Multidisciplinary approach generally means that specialized disciplines are applied in a study of real system. These disciplines provide partial analysis of the real system. These mono-disciplinary analyses are integrated to overall solution by integrating the solver who has basic multi-disciplines knowledge. In our case study are integrated four disciplines, namely, experimental physics, theoretical physics, mathematics and computer science.

3. Case Study – Modeling and Simulation of Artillery Projectile Trajectory Motion (APTM)

3.1 *Mathematical Model*

The motion of any object is described by Newton's vector equation,

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} \tag{1}$$

The net external force $\vec{F}$ includes all possible forces that may affect the motion of the object. For example, in a region where gravitational field permeates and the fluid medium offers resistance, the net force is given by

$$\sum \vec{F} = \vec{F}_g + \vec{F}_r, \tag{2}$$

where $\vec{F}_g$ is the gravitational force which, by assumption, is constant. $\vec{F}_r$ represents the fluid resistance that is proportional to some power n of the instantaneous velocity $\vec{v}$. Its magnitude, as defined empirically, is found to be

$$F = kv^n \tag{3}$$

where k is a proportionality constant that depends on the size and geometry of the object, as well as the properties of the medium. This proportionally constant k is commonly expressed in terms of the drag coefficient C_d, the density) of the air, and the cross-sectional area S of the object. The approximated fluid resistance, then, in vector form is

$$\vec{F}_r = -0.5 C_d \rho S v^n \tag{4}$$

For an object moving at a low speed, $n \approx 1$. That is, the resisting force is approximately proportional to the object's velocity. At a high speed, the resisting force increases proportionally with the square of the velocity of the object. As a result Newton's equations for vertical and horizontal motions become

$$m\frac{dv}{dt} = -kv^n - mg \tag{5}$$

$$m\frac{dv}{dt} = -kv^n \tag{6}$$

If $n \approx 1$, then by direct integration, the solutions to equations (5) and (6) are given by

$$v(t) = C_1 e^{-\left(k/m\right)t} - \frac{mg}{k}, \text{ and} \tag{7}$$

$$v(t) = C_2 e^{-\left(k/m\right)t} \tag{8}$$

where C_1 and C_2 are constants of integration that depends on certain initial conditions. At higher velocities ($n \approx 2$), similar analysis will show that equation (5) is satisfied by

$$v(t) = \left(\sqrt{mg/k}\right)\left[\frac{1 + Ce^{k/m\,t}}{1 - Ce^{k/m\,t}}\right] \tag{9}$$

Noting that *v(t)= dr/dt,* corresponding positions of the object can then be determined by integrating both sides of equations (7), (8), and (9) with respect to time. So far, the details of the dynamic analysis of moving objects in resistive media seemed mathematically overwhelming, and so a numerical simulation is performed, instead, by adapting the Euler scheme to calculate the positions and velocities of an object at various points in the trajectory.

3.1 *Methodology*

The curved flight of an artillery projectile subject to gravitational influence and effects of air drag is the dynamic model used in this study to generate the following vector equation of motion:

$$\frac{d^2\vec{r}}{dt} = \frac{1}{m}\vec{F}_r - \vec{g}\hat{y} \tag{10}$$

Using the fluid resistance expression in equation (4), and setting $n \approx 2$, equation (10) becomes

$$\frac{d^2\vec{r}}{dt} = -\left(\frac{0.5C_d\rho S}{m}\right)\left(\frac{d\vec{r}}{dt}\right)^2 - \vec{g}\hat{y} \tag{11}$$

This is a second order differential equation, which can be replaced by a system of two first order equations:

$$\vec{v}\,' = \frac{d\vec{v}}{dt} = \vec{a}\left(\vec{r}(t), \vec{v}(t)\right) \tag{12}$$

$$\vec{r}\,' = \frac{d\vec{r}}{dt} = \vec{v}(t) \tag{13}$$

By definition, equations (12) and (13) can also be written as

$$\frac{d\vec{v}}{dt} = \frac{\vec{v}(t+\tau) - \vec{v}(t)}{\tau} + \mathrm{O}(\tau) = \vec{a}\big(\vec{r}(t), \vec{v}(t)\big) \tag{14}$$

$$\frac{d\vec{r}}{dt} = \frac{\vec{r}(t+\tau) - \vec{r}(t)}{\tau} + \mathrm{O}(\tau) = \vec{v}(t) \tag{15}$$

where τ is called the time increment and *O*(τ) is the order of truncation error. In practical situations equations (14) and (15) are replaced by Forward Difference formulas:

$$\vec{a}_i = \frac{\vec{v}_{i+1} - \vec{v}_i}{\tau} \tag{16}$$

$$\vec{v}_i = \frac{\vec{r}_{i+1} - \vec{r}_i}{\tau} \tag{17}$$

Rearranging equations (16) and (17)

$$\vec{v}_{i+1} = v_i + \tau\vec{a}_i \tag{18}$$

$$\vec{r}_{i+1} = \vec{r}_i + \tau\vec{v}_i \tag{19}$$

Equations (18) and (19) are the equations used by the Euler method to find the numerical solutions to system equations (12) and (13). The positions and velocities of the baseball are continuously updated by r_{i+1} and v_{i+1}, respectively, and are recorded for subsequent analysis of the trajectory.

3.3 *Algorithm for Simulation*

The following details of the Euler algorithm are implemented using the MATLAB programming language, but can be adapted for other programming environments as well.

Step 1: Specify the known parameters of the model: projectile mass *m* and cross-sectional area *S*; the density ρ of the air, the drag coefficient C_d, and the gravitational acceleration *g*.

Step 2: Choose the time increment S; input initial values of position *r*, velocity *v*, and projection angle θ_0.

Step 3: Choose the maximum number of steps.

Step 4: Record the trajectory (i.e., [rx, ry]).

Step 5: Calculate the acceleration using equation (12)

Step 6: Calculate the new position and velocity using equations (14) and (15).

Step 7: Determine if rx ≤ 0. If true, then stop.

Step 8: Repeat steps 5 to 7 until the condition in step 7 is satisfied.

Step 9: Determine the maximum range and time of flight.

Step 10: Plot the trajectory

3.3 *Results and Discussion*

All computations are made using the following parameters: mass = 0.145 kg, diameter = 3.7 x 10^{-2} m, air density = 1.20kg/m^3, and drag coefficient = 0.35. Interestingly, at a low velocity of v = [10 10] m/s, the calculated range and time of flight based on the dynamic model agreed closely to the values predicted by the kinematic model. This fact is reinforced by the simulated parabolic trajectory of the projectile shown in Figure 2.

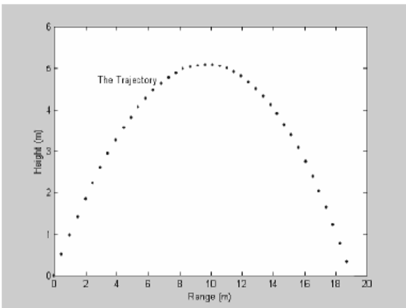

Fig. 2: Simulated baseball trajectory with $\vec{v}$ = [10 10], θ = 45^0.

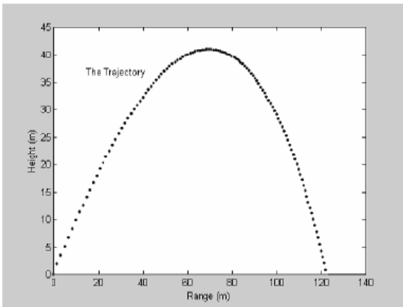

Fig. 3: Simulated baseball trajectory with $\vec{v}$ = [55 55], θ = 45^0.

This is expected of course since the drag force decreases also with the velocity v. However, at a high velocity v = [55 55] m/s, a discrepancy between the simulated and kinematic ranges is very noticeable. Air resistance increases rapidly with the square of the velocity and therefore forces the baseball to fall almost straight down. The simulated trajectory is shown in Figure 3. The computational technique used in the analysis of the flight of a baseball is indeed very useful and effective. Though the model equations do not include all the factors mentioned the results have been very satisfactory and even served well to convince us how close our model is to reality.

4. Conclusions

Modeling and computer simulation together with multidisciplinary approach provides new methodology of experimental research in current science. In this paper there has been discussed the case study of process of modeling and simulation of real experimental device - *Artillery Projectile Trajectory Motion (APTM)*. Step by step there is shown this process from formation of the mathematical model, creation of the simulation model in MATLAB with visualization and verification of the model.

References

[1] J A Sokolowski and C M Banks, *Principles of Modeling and Simulation – A Multidisciplinary Approach* (Wiley Publication, New Jersey, 2009).

[2] J Benacka, *International Journal of Mathematical Education in Science and Technology* **41(3)**, 373 (2010).

[3] S Hubalovsky, International Journal of Applied Mathematics and Formatics **4(4)**, 83 (2010).

[4] D Basmadjian, Chapman and Hall, *The Art of Modeling in Science and Engineering* (CRC Press, 1999).

[5] R L Woods and K L Lowrence, *Modelling and Simulation of Dynamic System* (Prentice - Hall, New Jersey, 1997).

[6] F R Giordano, M D Weir & W P Fox, *A First Course in Mathematical Modeling* (Thomson Brooks, Vikas Publishing House, USA, 3rd Edn , 2003).

[7] V.P. Singh, *System Modeling and Simulation* (New Age International(P) Limited, Publishers, New Delhi, 2009).

[8] GM Moss, DW Leeming and CL Farrar, *Military Ballistics: A Basic Manual* (Brassy's Ltd series, UK, London, 1995) Vol.-I.

[9] D E Carlucci and S S Jacobson, *Ballistics: Theory and Design of Guns and Ammunition* (CRC Press, Taylor & Francis Group, New York, 2007).