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Modeling Stiles Crawford Effect of the First Kind as Pupil Apodization

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Abstract: A beam entering a pupil from the edge elicits a poor visual response in comparison to an identical beam passing axially. This diminution in the effective brightness is a retinal phenomenon known as the Stiles-Crawford Effect of the first kind (SCE I). But in this study, modeling the effect as a pupil apodization, both in coherent and incoherent incident light and evaluating the modulation transfer function (MTF) of a periodic object of infinite cycles like a sine wave transmission profile has led to different degree of modification in the modulation of the retinal light distributions. While the incoherent light shows a modification of as high as 35 % in the modulation over a wide range of spatial frequencies, the modulation remains unmodified in coherent illumination. Various other apodization parameters are also taken to evaluate the MTF to understand its response to spatial frequencies under both coherent and incoherent illuminations. And it is found that for a large value of apodization parameter, the Gaussian character is lost and the modulation assumes a constant value of unity for all values of spatial frequencies for incoherent illumination. But for coherent entering light the SCE I apodized human eye does not show any modification in the modulation thereby pointing to a possibly governing role the formation of an interference pattern on the retina assumes in regulating retinal light distributions besides the traditional influence the Stiles Crawford effect of the first kind has hitherto offered.

1. Introduction

A beam of light stimulates the retina weakly when its entry to the pupil is gradually shifted from the centre towards the edge. This is manifested as a reduction in visibility, also known as Stiles-Crawford effect of the first kind (SCE I) [1]. Mathematically it can be expressed either as $\eta = 10^{-\rho_{10} r^2}$ or $\eta = e^{-\rho_e r^2}$, where *r* is the distance in the entrance pupil from the origin of the function and η is the visibility [2]. The coefficients (of directionality that measures the width of the pupil apodization) ρ_e and ρ_{10} are related by $\rho_e = \ln 10 \rho_{10} = 2.3\rho_{10}$. The means and standard deviations [3] for ρ_{10} are: horizontal meridian, 0.048±0.013 mm^{-2} ; vertical meridian, 0.053±0.012 mm^{-2}. In spite of its retinal origin SCE I has been regarded as pupil apodization [4-7]. Actually the twin techniques of modeling the Stiles-Crawford effect of the first kind as pupil apodization [8] and the wave guiding of light in the photoreceptors are theoretically employed to compute the retinal light distributions [9-12]. In this paper the apodization approach is adhered to.

2. Images of periodic sine -wave targets

The object amplitude transmission in a sinusoidal grating object can be mathematically expressed as [13-14]

$$A(x, y) = a_1 + a_2 \cos(\omega x) \tag{1}$$

where a_1 is the average amplitude in the object and a_2 is the modulation in the object amplitude, ω is the angular spatial frequency in reduced units, that is, $\omega = \frac{2\pi}{p}$ and x, y are the horizontal and vertical coordinates of space.

The Fourier transform of Eq. (1) leads to

$$a(u,v) = \iint_{-\infty}^{\infty} A(x, y) e^{-i(ux+vy)} dx\, dy$$

$$= \iint_{-\infty}^{\infty} \left[a_1 + a_2 \cos(\omega x)\right] e^{-(ux+vy)} dx\, dy \tag{2}$$

The evaluation of the above integral calls for the use of the two dimensional Dirac delta function defined as

$$\delta(p,r) = \int\int_{-\infty}^{\infty} e^{-i(pq+rs)} dq\, ds \tag{3}$$

Thus Eq. (2) reduces to

$$= a_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(ux+vy)} dx\, dy + a_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cos(\omega x) e^{-(ux+vy)} dx\, dy$$

$$= a_1 \delta(u,v) + \frac{a_2}{2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [e^{-i[(u-\omega)x+vy]} + e^{-i[(u+\omega)x+vy]}] dx\, dy$$

$$= a_1\,\delta(u,v) + \frac{a_2}{2}[\delta(u-\omega,v) + \delta(u+\omega,v)]$$

Given that the Fourier transform of the input to the pupil is $a(u,v)$, pupil exit function will be $f(u,v)\,a(u,v) = a'(u,v)$ for f being the amplitude transfer function which we approximate as follows [3]:

$$f(u,v) = \begin{cases} e^{\frac{-(u^2+v^2)}{\sigma^2}} & \text{if } u^2+v^2 < K \\ 0 & \text{otherwise} \end{cases}$$

where K = 1 for diffraction-limited coherent imaging systems, and K = 2 for diffraction –limited incoherent imaging systems [4, 15-17] for apodization parameter σ =3.086 [3]

$$a'(u,v) = e^{\frac{-(u^2+v^2)}{\sigma^2}}\left[a_1\,\delta(u,v) + \frac{a_2}{2}\{\delta(u-\omega,v) + \delta(u+\omega,v)\}\right] \tag{4}$$

The image amplitude distribution at the exit pupil is obtained by the inverse Fourier transform of the spectrum $a'(u,v)$. Thus

$$A'(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a'(u,v)\,e^{i(ux+vy)}du\,dv$$

$$A'(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)[a_1\delta(u,v) + \frac{a_2}{2}\{\delta(u-w,v) + \delta(u+\omega,v)\}]\,e^{i(ux+vy)}du\,dv$$

$$= a_1 + a_2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{[\delta(u-\omega,v) + \delta(u+\omega,v)]}{2}\,f(u,v)\,e^{i(ux+vy)}du\,dv$$

$$= a_1 + a_2\,f(\omega,0)\cos(\omega x)$$

$$A'(x,y) = a_1 + a_2\,f(\omega,0)\cos(\omega x) \tag{5}$$

Finally, the image energy distribution will be given by the squared modulus of above as

$$|A'(x,y)|^2 = [a_1 + a_2\,f(\omega,0)\cos(\omega x)]^2 \tag{6}$$

ωx can vary from 0 to π to compute maximum and minimum image energy distribution.

The modulation in the image (for unit contrast in the object) can be defined as

$$M = \frac{|A'(x,y)|^2_{\max} - |A'(x,y)|^2_{\min}}{|A'(x,y)|^2_{\max} + |A'(x,y)|^2_{\min}} \tag{7}$$

$$|A'(x,y)|^2_{\max} = \left[a_2 + a_2\, e^{-\frac{\omega^2}{\sigma^2}} \right]^2$$

$$= a_1^2 + a_2^2\, e^{-\frac{2\omega^2}{\sigma^2}} + 2a_1 a_2\, e^{-\frac{\omega^2}{\sigma^2}}$$

$$|A'(x,y)|^2_{\min} = \left[a_1 - a_2\, e^{-\frac{\omega^2}{\sigma^2}} \right]^2$$

$$= a_1^2 + a_2^2\, e^{-\frac{2\omega^2}{\sigma^2}} - 2a_1 a_2\, e^{-\frac{\omega^2}{\sigma^2}}$$

Thus, taking $a_1 = a_2 = \frac{1}{2}$ we get

$$M = \frac{4a_1 a_2\, e^{-\frac{\omega^2}{\sigma^2}}}{2a_1^2 + 2a_2^2\, e^{-\frac{2\omega^2}{\sigma^2}}} = \frac{2}{1 + e^{-\frac{2\omega^2}{\sigma^2}}} = \frac{2e^{-\frac{\omega^2}{\sigma^2}}}{e^{-\frac{\omega^2}{\sigma^2}}\left(e^{\frac{\omega^2}{\sigma^2}} + e^{-\frac{\omega^2}{\sigma^2}} \right)} = \operatorname{sech}\frac{\omega^2}{\sigma^2}$$

$$M_{coh} = \operatorname{sech}\frac{\omega^2}{\sigma^2} \tag{8}$$

As the incoherent imaging is linear in image energy distribution, rather than amplitude the image energy distribution and the corresponding modulation equations for incoherent illumination will be given by

$$I(x,y)_{incohernt} = a + b\, e^{-\frac{\omega^2}{\sigma^2}} \cos(\omega x) \tag{9}$$

where *a* is average intensity and *b* is modulation.

$$M_{inc} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$I_{\max} = a + b e^{-\frac{\omega^2}{\sigma^2}}$$

$$I_{\min} = a - b e^{-\frac{\omega^2}{\sigma^2}}$$

Taking average irradiance (or intensity) and the modulation in the object energy distribution as equal, we get the contrast in the image as

$$M_{inch} = e^{-\frac{\omega^2}{\sigma^2}} \tag{10}$$

3. Results and Discussion

The retinal light distributions in the images of periodic sine wave targets, formed by a human eye apodized with SCE-I under coherent and incoherent illumination was computed using Eq.(6) and Eq (9) respectively. Eq. (8) is used to find out the modulations in the images of sinusoidal grating targets for coherent illumination. Similarly modulation is evaluated for the incoherent illumination using Eq. (10). The values are plotted from Fig.1 to Fig. 7.The cut-off frequency is the highest spatial frequency that a normal human eye could resolve if it had perfect optics. So, the fineness of the retinal sampling array imposes a 75 Hz cut-off on sine waves that can be delivered to the retina using incoherent light. And for coherent light this will be lower, i.e., 37.5 cycles/deg [16-17,21-23]. All the computations are tabulated in Table 01.

From Fig.1, it is evident that for an apodization parameter of σ = 1.0, the modulation decreases monotonically with the increase of the spatial frequency in case of incoherent illumination. But for coherent illumination the variation is not pronounced. By changing the apodization parameter to σ = 2.0, we see from Fig.2 that the modification in the modulation for incoherent illumination continues unhindered while that for coherent light it almost remains unmodified. σ = 3.086 is the value of the apodization parameter for a human eye with Stiles Crawford effect of the first kind. This value is important for our study and we see from Fig. 3 that the modulation registers a change of 35 % in the modulation over the entire range of spatial frequencies. But the modulation does not get modified for coherent light.

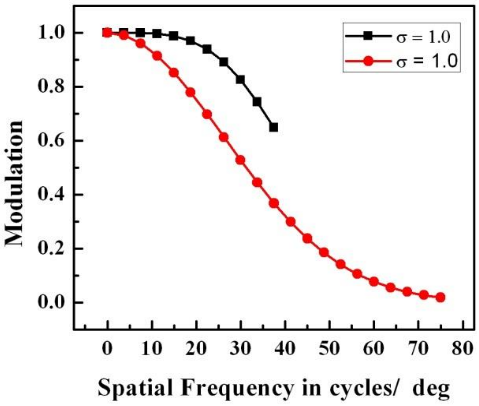

Fig.1: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter σ =1.0

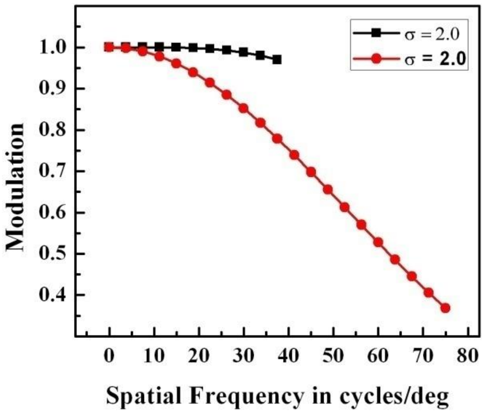

Fig.2: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter σ =2.0

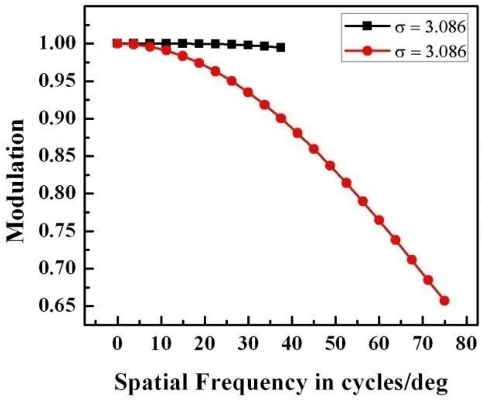

Fig.3: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter σ =3.086

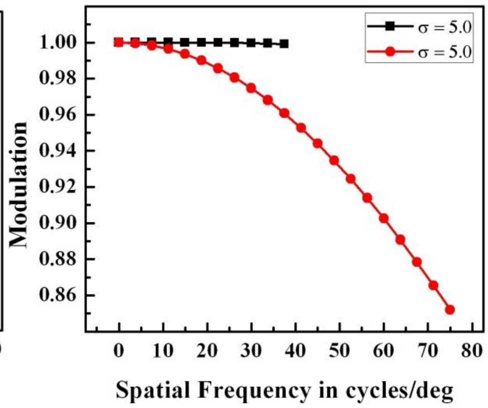

Fig.4: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter σ =5.0

By increasing the apodization parameter to σ = 5.0, even the changes in modulation decreases in case of incoherent illumination also. (Fig. 4)

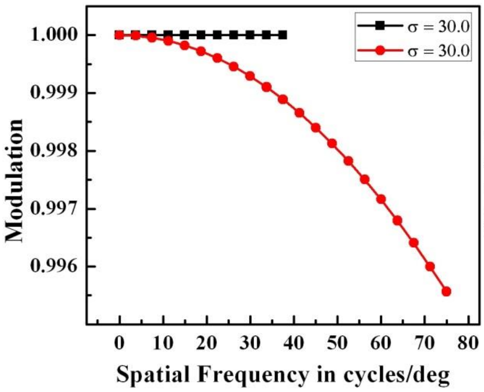

Fig.5: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter σ =30.0

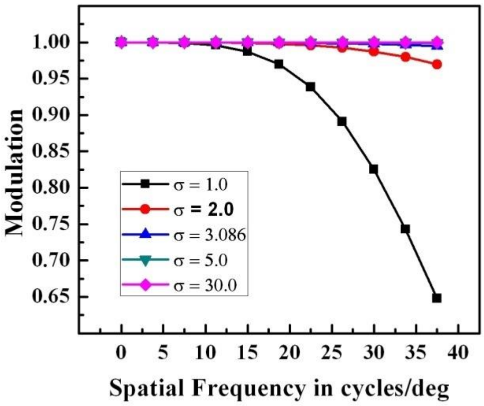

Fig.6: Variation of modulation with spatial frequency for coherent illumination for apodization parameters σ =1.0, 2.0, 3.086, 5.0 and 30.0

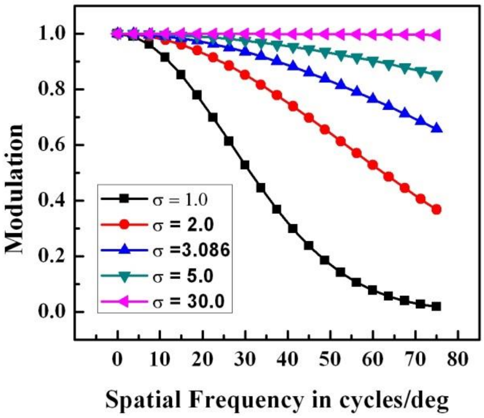

Fig.7: Variation of modulation with spatial frequency for incoherent illumination for apodization parameters σ =1.0, 2.0, 3.086, 5.0 and 30.

And with a large apodization of σ = 30.0 the Gaussian character is lost and the modulation even for incoherent illumination hovers around the constant value of unity. (Fig. 5)

Fig. 6 depicts the overall response of modulation to spatial frequencies for different apodization parameters when the entering light is coherent. Again the modulation is modified except for σ = 1.0. And Fig. 7 depicts the overall response of modulation to spatial frequencies for different apodization parameters when the entering light is incoherent. There are various degrees of modification in modulation for all apodization parameters except σ = 30.0

This point to the following possible mechanism. When a pair of symmetrically placed coherent beams entering from the opposite edges of the pupil forms an interference pattern of unit contrast on the retina, it gives rise to a Poynting vector normal to the image plane. But when the entering beams are devoid of spatial coherence, they keep their individual Poynting vectors uncoupled and thus oblique to the image plane. Hence when the Poynting vector has no oblique incidence the two coherent beams in effect corresponds to the peak of the pupil's Stiles-Crawford function, and brightness remains intact manifesting itself in a nil-modification in the modulation evaluated using a sinusoidal grating target. But with the two beams being incoherent, they fail to interfere, and each gives rise to its own obliquely pointed Poynting vector, eventually showing reduction in brightness as manifested in a modification in the modulation [7, 18,24].

4. Conclusion

When the entering beam is totally spatially coherent, the retinal light distributions is computed by modeling SCE I as a pupil apodization and choosing a sinusoidal grating object as a test target we have reached some important results regarding the visual performance of a human eye.

The modulation's inability in showing the expected modification in coherent illumination for a human eye where the SCE I is modeled as a pupil apodization points to a more deeper fact in working, that is, the retina prefers to respond to the more immediate stimulus of an interference pattern than to the pupil entry points of the entering beam hitherto governed traditionally by Stiles-Crawford effect of the first kind only. Recent experiments [7,19-20] have also validated such line of arguments and findings. In a proceeding paper to be published it is shown that the coherence in the entering beam can be used as an optical tool to regulate the spatial distribution of photon absorption in the photoreceptors of a human eye [18].

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Table 1. Computation of Modulation for different apodization parameters over the entire range of spatial frequencies under both coherent and incoherent incident illumination

	$M_{coh.}$					$M_{incoh.}$				
ω in c/deg	σ =1.0	σ =2.0	σ =3.086	σ =5.0	σ =30.0	σ =1.0	σ =2.0	σ =3.086	σ =5.0	σ =30.0
3.75	0.99995	0.9999969	0.9999994	0.9999999	1	0.9900498	0.9975031	0.9989505	0.9996001	0.9999889
7.50	0.9992005	0.99995	0.9999912	0.9999987	1	0.9607894	0.9900498	0.9958086	0.9984013	0.9999556
11.25	0.9959636	0.9997469	0.9999553	0.9999935	1	0.9139312	0.9777512	0.9905941	0.9964065	0.9999
15.00	0.9873351	0.9992005	0.9998589	0.9999795	1	0.8521438	0.9607894	0.9833396	0.9936204	0.9998222
18.75	0.9695436	0.99805	0.9996555	0.99995	1	0.7788008	0.9394131	0.9740904	0.9900498	0.9997223
22.50	0.938524	0.9959636	0.9992859	0.9998963	0.9999999	0.6976763	0.9139312	0.9629039	0.9857032	0.9996001
26.25	0.8908914	0.9925435	0.9986778	0.999808	0.9999999	0.6126264	0.8847059	0.949849	0.9805908	0.9994557
30.00	0.8251597	0.9873351	0.9977461	0.9996724	0.9999997	0.5272924	0.8521438	0.9350055	0.9747249	0.9992891
33.75	0.7427307	0.9798414	0.9963938	0.9994753	0.9999996	0.4448581	0.8166865	0.918463	0.9681193	0.9991004
37.50	0.6480543	0.9695436	0.9945122	0.9992005	0.9999994	0.3678794	0.7788008	0.9003204	0.9607894	0.9988895
41.25	0.5476928	0.9559287	0.9919824	0.9988299	0.9999991	0.2981973	0.7389685	0.8806848	0.9527526	0.9986565
45.00	0.4486696	0.938524	0.9886762	0.9983434	0.9999987	0.2369278	0.6976763	0.8596701	0.9440275	0.9984013
48.75	0.3568879	0.9169359	0.9844584	0.9977195	0.9999982	0.1845195	0.6554063	0.8373964	0.9346343	0.998124
52.50	0.276236	0.8908914	0.9791888	0.9969346	0.9999976	0.1408584	0.6126264	0.8139886	0.9245945	0.9978246
56.25	0.2084824	0.8602752	0.9727253	0.9959636	0.9999969	0.1053992	0.5697828	0.7895752	0.9139312	0.9975031
60.00	0.153691	0.8251597	0.9649269	0.9947799	0.999996	0.0773047	0.5272924	0.7642872	0.9026684	0.9971596
63.75	0.1108102	0.7858198	0.955658	0.9933553	0.9999948	0.0555762	0.4855369	0.7382571	0.8908315	0.996794
67.50	0.0782078	0.7427307	0.9447925	0.9916603	0.9999935	0.0391639	0.4448581	0.7116175	0.8784467	0.9964065
71.25	0.0540641	0.6965451	0.9322183	0.9896641	0.999992	0.0270518	0.4055545	0.6845001	0.8655415	0.9959969
75.00	0.036619	0.6480543	0.9178421	0.9873351	0.9999901	0.0183156	0.3678794	0.6570348	0.8521438	0.9955654