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Physical and Mathematical Aspects of Statistical Energy Analysis

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Abstract : Vibration is a ubiquitous problem for design aspects of various mechanical systems and similar structures composed of such elements as plates and beams. The most famous method in theoretical modelling and analyzing high frequencies is Statistical Energy Analysis (SEA) method is a way of studying dynamical system. This method is "energy-based" in contrast to classical methods that are based on quantities such as force and displacement. This paper discusses the physical and mathematical aspects of SEA and listing some areas where SEA has been applied successfully.

Keywords : Vibration, Dynamical System, Acoustics, Wave, Entropy

1. Introduction

1.1 *Basic Concepts and History of Sea*

The investigation of the governing equations of vibrating structures and acoustics began in the eightieth century with the study of string vibration by d'Alembert and beams by Euler and Bernoulli. During the nineteenth century and the early of the twentieth century, the governing equations for almost all simple vibrating systems have been derived, plates by Germain, beams with shear effect by Timoshenko, shells by Donnell, high order plate theory by Mindlin but with many other contributors such as Helmholtz, Kirchhoff, Rayleigh, Sommerfeld, Love, Lamb and many others. The famous Rayleigh's treatise, The Theory of Sound, published in 1894, gives a precise idea of what was the state of the art at the end of the ninetieth century.

Great number of physical structures is subjected to vibration and acoustic excitation at high frequencies. The most theoretical modelling approach for analyzing high frequencies is Statistical Energy Analysis (SEA), which is “energy-based” approach in contrast to classical approach. It is, also called Statistical vibroacoustics, which is the science of structural waves, acoustical waves and their interaction in the field of science and engineering, is born from the application of statistical physics concepts to the study of random vibration in mechanical and acoustical systems. Presented in classical textbooks, SEA is usually divided into the study of propagation of waves, natural modes, radiation of sound, sound transparency and structural response. This science is at the interface of acoustics and elastodynamics that is the theories of propagation of small perturbations in fluids and solids. As such, SEA focuses on the coupling between structural and acoustical waves.

Traditionally, in analysis of mechanical vibrations, the lowest modes are usually of most interest because these modes tend to produce the greatest displacement response. But while designing large and lightweight structures, it is imperative to account for high frequency broadband loads to predict fatigue, failure or noise emission. The SEA proves to be an effective method to predict high frequency loads. Since its formulation, SEA has been widely used in a growing number of applications. It has also been successful in predicting the average vibrational amplitudes and sound pressures in space vehicles, airplanes, ships, buildings, large machines, etc.

SEA is a way of studying dynamical systems. The development of SEA by RH Lyon emerged to predict the vibrational response to rocket noise of satellite launch vehicles and their payloads in the early 1960’s. Since then its use has become widespread and it is currently used in a multitude of different applications ranging from shipboard noise to automobile acoustics. SEA is well suited to predicting the response of complex structural-acoustic systems over a large frequency range (typically 50 - 20,000 Hz), and it can be used to model both random and tonal sources.

SEA is a method for predicting the transmission of sound and vibration through complex structural acoustic systems. The method is particularly well suited for quick system level response predictions at the early design stage of a product, and for predicting responses at higher frequencies. In SEA a system is represented in terms of a number of coupled subsystems and a set of linear equations are derived that describe the input, storage, transmission and dissipation of energy within each subsystem. The parameters in the SEA equations are typically obtained by making certain statistical assumptions about the local dynamic properties of each subsystem (similar to assumptions made in room acoustics and statistical mechanics). These assumptions significantly simplify the analysis and make it possible to analyze the response of systems that are often too complex to analyze using other methods (such as finite element and boundary element methods).

SEA method is based on energy dissipation. "Statistical" means, that the variables are extracted from statistical population and all results are expected values. The main idea of SEA method is that one has to divide analyzed structure into "subsystems". All energy analysis is done between those subsystems. What is the subsystem? It's a part or physical element of a structure (system) being analyzed. SEA is a probabilistic analysis tool to determine the global vibrational energies of complicated systems. Though the vibrational modes of structures could to be predicted computationally, the size of the models and the computational speed was discouraging to the engineers as it could predict only a few of the lowest order modes.

The underlying theory of SEA is based on the principles of statistical mechanics and conservation of energy and there are many parallels between an SEA analysis and a thermal analysis. The basic insight of conventional SEA is that, under most circumstances, energy in the form of vibration behaves the same way as energy m the form of heat it diffuses from the 'hotter' places to the 'cooler' ones at a rate proportional to the difference of 'temperature', the constant of proportionality being a measure of 'thermal conductivity'. This paper discusses the basic ideas, physical aspects, and mathematical aspects with its analysis of SEA and by listing some typical areas where SEA has been used very successfully.

2. Physical Aspects of SEA

The concepts of vibrational temperature, vibrational heat and vibrational entropy and their relationships exactly match with the classical definitions and relationships in thermodynamics. Statistical energy analysis (SEA) is born from the application of statistical physics concepts to vibroacoustics. Elaborated during the sixties, this theory lies on a simple idea: When the number of modes in vibroacoustical systems is so large that the solving of governing equations becomes unrealistic, it is preferable to give up a deterministic description of the system and to adopt a statistical point of view.

As an approach to the study of mechanical vibrations, statistical energy analysis (SEA) has found new applications. The name SEA is coined to emphasize the essential feature of the approach: "*Statistical*" indicates that the dynamical systems under study are presumed to be drawn from statistical populations or ensembles in which the distribution of the parameters is known. "*Energy*" denotes the primary variable of interest. "*Analysis*" is used to underscore the fact that SEA is a general framework of methods rather than a particular technique.

SEA is a modelling procedure for the theoretical estimation of the dynamic characteristics of, the vibrational response levels of, and the noise radiation from complex, resonant, built-up structures using energy flow relationships. These energy flow relationships between the various coupled subsystems (e.g. plates, shells, etc.) that comprise the built-up structure have a simple thermal analogy. Many random noise and vibration problems cannot be practically solved by classical methods and SEA therefore provides a basis for the prediction of average noise and vibration levels particularly in high frequency regions where modal densities are high. SEA has evolved over the past two decades and it has its origins in the aerospace industry. It has also been successfully applied to various industry applications, and it is now being used (i) as a prediction model for a wide range of industrial noise and vibration problems, and (ii) for the subsequent optimization of industrial noise and vibration control.

Lyon's book on the general applicability of SEA to dynamical systems was the first serious attempt to bring the various aspects of SEA into a single volume. It is a useful starting point for anyone with a special interest in the topic. There have been numerous advances in the subject since the publication of Lyon's book, and Fahy and Hodges and Woodhouse discuss some of these advances in review papers.

As such, SEA can be classified as a "node-connector" type of modeling, similar to network analysis in thermal, electrical and fluid flow problems. In this case, the SEA "nodes" represent the reverberant energy level of resonant mode groups in each substructure region and the "connectors" represent the energy flow paths between nodes. Most older generation SEA codes use this network paradigm extensively as the basis for modeling.

Noise and vibration are often treated separately in the study of dynamics, and it is sometimes forgotten that the two are interrelated - i.e. they simply relate to the transfer of molecular motional energy in different media (generally fluids and solids respectively). The concept of wave-mode duality is generally convenient for engineers to think of noise in terms of waves and to think of vibration in terms of modes. A fundamental understanding of noise, vibration and interactions between the two therefore requires one to be able to think in terms of waves and also in terms of modes of vibration.

For most SEA applications, it is assumed that the majority of the energy flow between subsystems is due to resonant structural or acoustic modes - i.e. SEA is generally about energy or power flows between different groups of resonant oscillators, although some work has been done on extending it to non-resonant systems.

Woodhouse who discusses a very simple thermal analogy can find an excellent conceptual introduction to SEA in a paper - i.e. vibrational energy is analogous to heat energy. Heat energy flows from a hotter to a cooler place at a rate proportional to the difference of temperature. The constant of proportionality in this instance is a measure of thermal conductivity. As a simple example, Woodhouse considers two identical elements, one of which is supplied by heat from some external source. The model is illustrated in Figure 1. The two parameters of primary importance are the radiation losses and the degree of coupling via the thermal conductivity link. In practice, situations of high or low radiation losses and high or low thermal conductivity can arise. High thermal conductivity implies a strong coupling link between the two elements, and low thermal conductivity suggests a weak-coupling link. Four possible situations can arise. These situations are illustrated schematically in Figure 2.

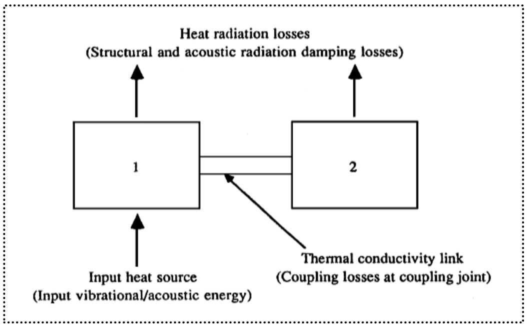

Fig. 1: Thermal – Vibration/Acoustic Analogy

There is an analogy between the thermal model and certain parameters associated with noise and vibration because the flow of vibrational energy in a structure (or noise in an acoustic volume) behaves in the same way as the flow of heat. Provided that there are sufficient resonant structural or acoustic modes within a frequency band of interest, the mean modal energy can be regarded as being equivalent to a measure of temperature. The modal density (number of modes per hertz) is analogous to the thermal capacity of the thermal model, the internal loss factors (damping) are analogous to the radiative losses of the thermal model, and the coupling loss factors (a measure of the strength of the mechanical coupling between the subsystems) are analogous to the thermal conductivity links between the various elements in the thermal model. For two coupled subsystems, Figure 2 shows how the mean-square vibrational levels depend on damping and coupling loss factors, and how mean-square temperature levels depend upon radiation and thermal conduction. Consider again the two-subsystem example in Figure 1. If this were a structural system then the input would be some form of vibrational energy, the radiation losses would correspond to internal losses due to structural and acoustic radiation damping, and the conductivity link would be associated with coupling losses at the coupling joint between the two subsystems.

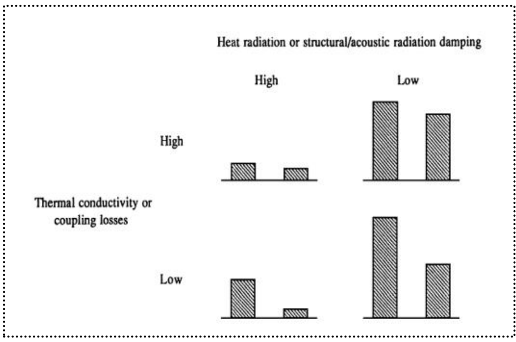

Fig. 2: Mean-square temperatures or vibrations energies for various energy loss combinations

Now, assume that (i) the subsystems are strongly coupled; (ii) only subsystem 1 is directly driven; (iii) Subsystem 1 is lightly damped; (iv) Subsystem 2 are heavily damped, and that one wishes to minimize the vibrational levels transmitted to subsystem 2. Vibration isolation between the two subsystems would not be effective by itself because the vibrational levels in subsystem 1 would rise to a possibly unacceptable level since it is lightly damped (vibration isolation would prevent the vibrational energy from flowing to the more heavily damped subsystem where it could be dissipated). Alternatively, if the vibration isolator was removed and damping treatment was added to subsystem 1 instead, a significant amount of vibrational energy would flow to subsystem 2 because of the strong coupling, and because both subsystems are heavily damped they would both have approximately the same amount of energy. However, if subsystem 1 was damped and vibration isolation was provided between the two subsystems to reduce the coupling link, then most of the energy generated in subsystem 1would be dissipated at source. This simple qualitative example illustrates how an analysis based upon S.E.A. procedures can provide a very powerful tool for the parametric study of energy flow distributions between coupled subsystems for the purposes of optimizing noise and vibration control.

The SEA modelling procedures require information about three structural parameters: (i) the modal densities of the various subsystems, (ii) the internal loss factors of the various subsystems, and (iii) the coupling loss factors of the various coupling joints. The modal density defines the number of modes per unit frequency, the internal loss factor is associated with energy lost by structural damping and acoustic radiation damping, and the coupling loss factor represents the energy lost by transmission across a discontinuity such as a flange, a step change in wall thickness, or a structure–acoustic volume interface. Two specific situations arise with regard to the interpretation of modal densities. When there are numerous modes in a frequency band, if the individual modal peaks can be clearly identified, the modal overlap is defined as being weak – this is often the case for lightly damped structural components. If the individual modal peaks cannot be clearly identified, the modal overlap is defined as being strong – this is typically the case for reverberant sound fields. It should be clear by now that the breaking up of a system into appropriate subsystems is a very important first step in SEA.

2.1 *Important Parameters in SEA*

There are four essential parameters in the study of SEA: (a) the damping loss factor, (b) the coupling loss factor, (c) the power (input, dissipated, transmitted) and (d) number of modes per frequency band. The damping loss factor relates to the power dissipated in a subsystem. To experimentally determine the damping loss factor, it needs to be spatially averaged for each frequency band. The damping loss factor can be measured by various methods, for instance, the power injection method that is performed by applying a known power input. The coupling loss factor relates to the energy flow between subsystems. It is defined as the fraction of energy that is transmitted from one subsystem to another. While dealing with structures, the coupling loss factor is proportional to the transmission coefficient that depends upon the orientation, thickness and material properties of the structure. In acoustics, the coupling loss factor is proportional to the radiation efficiency. To experimentally determine the coupling loss factor within the subsystems, one of the subsystems should be more damped than the other. The damping loss factor of the other subsystem should be known. One of the subsystems is directly excited during the experiment. The reaction of both the subsystems must be evaluated to determine the energy in each subsystem. The power flow from one subsystem to another can be evaluated once you calculate all the loss factors which are dependent on the dimensions, material properties of the subsystems, and the energy transfer from one to another. The net power flow can then be calculated knowing the individual power flows for all subsystems. The fourth parameter is the number of modes per frequency band as mentioned that is the number of modes per the evaluated frequency band valid for both, the constant bandwidth and the octave band. Modal density is another important parameter, which emerges stating the number of modes per frequency bandwidth.

2.2 *Limitations and Assumptions in SEA*

SEA has limitation in accuracy at the lower frequency ranges; generally below 200-400 Hz. SEA cannot predict excitation at specific or narrow band frequencies. Due to the average frequency responses at a frequency band, it is incapable to predict modes or mode shapes of the system. It does not render information on local distribution vibration level within the subsystems. It is unable to give information about the spatial distribution of the field variables within each subsystem. Along with the limitations, there are various assumptions made while performing the SEA. The coupling between the subsystems is assumed to be linear and conservative. The resonant modes in a particular frequency band are assumed to have the same amount of energy. In addition, the damping loss factor is assumed to be equal for all modes in any particular frequency band. The damping should not be too low or too high. Homogeneity of the subsystems is imperative to yield valid vibrational level calculations. Also, sound fields have to be assumed to be reverberant and diffuse. The transmission of power from one subsystem to another subsystem is due to the existing resonant modes in the frequency band. Also the power flow within the subsystems varies proportionally to their energy level.

The procedures of SEA can be thought of as the modelling of elastic mechanical systems and fluid systems by subsystems, each one comprising groups of multiple oscillators, with a probabilistic description of the relevant system parameters. The analysis is thus about the subsequent energy flow between the different groups of oscillators. The procedures are based upon several general assumptions, namely:

(i) There is linear, conservative coupling (elastic, inertial and gyrostatic) between the different subsystems;

(ii) The energy flow is between the oscillator groups having resonant frequencies in the frequency bands of interest;

(iii) The oscillators are excited by broadband random excitations with uncorrelated forces (i.e. not point excitation), which are statistically independent – hence there is modal incoherency, and this allows for a linear summation of energies;

(iv) There is equipartition of energy between all the resonant modes within a given frequency band in a given subsystem;

(v) The principle of reciprocity applies between the different subsystems;

(vi) The flow of energy between any two subsystems is proportional to the actual energy difference between the coupled subsystems whilst oscillating - i.e. the flow of energy is proportional to the difference between the average coupled modal energies.

3. Mathematical Aspects of SEA Model

Let us consider a simple example of two subsystems as shown in Figure 3 where there are two subsystems: Subsystem 1 and Subsystem 2. The arrow pointing towards the subsystems indicate the power received by the subsystems. While the arrows pointing away from the subsystems indicate the power lost from the subsystems. The arrows pointing within the subsystems indicate the exchange of power between the subsystems. W_1 and W_2 show the power entering the Subsystems 1 and 2, respectively, whereas W_{1d} and W_{2d} indicate the power dissipated by the Subsystems 1 and 2, respectively. W_{12} shows the power the transfer of power from Subsystem 1 to Subsystem 2, and similarly W_{21} shows the transfer of power from Subsystem 2 to Subsystem 1.

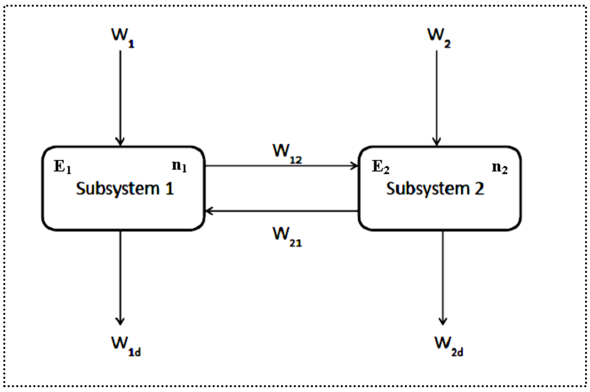

Fig. 3: A Two Subsystems SEA Model

The dissipated power at central angular frequency of ω in the subsystems can be shown by the following equations,

$$W_{1d} = \omega\eta_{1d} E_1 \tag{1}$$

$$W_{2d} = \omega\eta_{2d} E_2 \tag{2}$$

where η_{1d} and η_{2d} are the internal/damping loss factors of Subsystems 1 and 2 receptively. E_1 and E_2 are the total vibrational or acoustic energy of the modes at frequency *f*.

The net power transmitted and the modal relationship between subsystems can be expressed as,

$$W_{12} = \omega\eta_{12} E_1 \tag{3}$$

$$W_{21} = \omega\eta_{21} E_2 \tag{4}$$

$$\frac{\gamma}{n_1} = \omega\eta_{12} \text{ and } \frac{\gamma}{n_2} = \omega\eta_{21} \tag{5}$$

Thus, $$n_1\eta_{12} = n_2\eta_{21} \tag{6}$$

where η_{12} and η_{21} are the coupling loss factors between the Subsystems 1 and 2, n_1 and n_2 are the modal densities of the Subsystems and γ is the constant of proportionality.

The SEA calculation is based on energy flow equilibrium; hence the power balances for the two subsystems can be given by,

$$W_1 + W_{21} = W_{12} + W_{1d} \tag{7}$$

$$W_2 + W_{12} = W_{21} + W_{2d} \tag{8}$$

By using Equations (1), (2), (3) and (4), we have,

$$W_1 = \omega[(\eta_{12} + \eta_{1d})E_1 - \eta_{21}E_2] \tag{9}$$

$$W_2 = \omega[(\eta_{21} + \eta_{2d})E_2 - \eta_{12}E_1] \tag{10}$$

The power balance equation for the two subsystems can be expressed in matrix form as,

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \omega \begin{bmatrix} (\eta_{1d} + \eta_{12}) & -\eta_{21} \\ -\eta_{12} & (\eta_{2d} + \eta_{21}) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{11}$$

The generalized form of the power balance equation with n number of subsystems will be

$$\begin{bmatrix} W_1 \\ W_2 \\ \cdot \\ \cdot \\ \cdot \\ W_n \end{bmatrix} = \begin{bmatrix} \eta_1 & -\eta_{21} & \cdot & \cdot & \cdot & \eta_{n1} \\ -\eta_{12} & \eta_2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\eta_{1n} & \cdot & \cdot & \cdot & \cdot & \eta_n \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_n \end{bmatrix} \tag{12}$$

where η_i stands for the total loss factor of the i^{th} system which is the summation of the damping loss and the coupling loss factors and in general can be stated as

$$\eta_i = \eta_{id} + \sum_{j=1, j\neq 1}^{n} \eta_{ij} \tag{13}$$

where n is the number of subsystems and the subscript i and j represent the identities of the subsystems.

For the steady-state power balance equations for the two groups of oscillators from Equations (9) and (10) are

$$\omega E_1\eta_{1d} + \omega E_1\eta_{12} - \omega E_2\eta_{21} = W_1 \tag{14}$$

$$\omega E_2\eta_{2d} + \omega E_2\eta_{21} - \omega E_1\eta_{12} = 0 \tag{15}$$

The steady-state energy ratio between the two groups of oscillators can be obtained from equation (15). It is

$$\frac{E_2}{E_1} = \frac{\eta_{12}}{\eta_{2d} + \eta_{21}} \tag{16}$$

Equation (16) is a very important conceptual equation. It illustrates how energy ratios between coupled groups of oscillators can be obtained from the internal loss and coupling loss factors. Furthermore, if the input energy to Subsystem 1 is known, the output energy from Subsystem 2 can be readily estimated. By substituting Equation (6) into Equation (16), one gets

$$\frac{E_2^*}{E_1^*} = \frac{\eta_{21}}{\eta_{2d} + \eta_{21}} \tag{17}$$

where $E_1^* = E_1/n_1$ and $E_2^* = E_2/n_2$. For the special case of two coupled oscillators, rather two coupled groups of oscillators, the modal densities n_1 and n_2 are both equal to unity, hence $E_1^* = E_1$ and $E_2^* = E_2$.

Two important points, which draw an analogy with the thermal example, can be made in relation to Equation (17). Firstly, if $\eta_{2d} << \eta_{21}$ then $E_2^*/E_1^* \to 1$. This suggests that additional damping to Subsystem 2 will be ineffective unless η_{2d} can be brought up to the same level as η_{21}. Secondly, E_2^* is always less than E_1^* since η_{21} has to be positive. When $E_2^*/E_1^* \to 1$ there is equipartition of energy between the two groups of oscillators.

With knowledge of the modal densities and internal loss factors of two different subsystems, and the coupling loss factors between the subsystems, one can readily estimate the energy flow ratios. Alternatively, information could be obtained about the internal loss and coupling loss factors from the total energies of vibration and the modal densities.

Equation (17) can be re-written in terms of the total energies of vibration, E_1 and E_2, of the two groups of oscillators. Using the consistency relationship (Equation 6),

$$\frac{E_2}{E_1} = \frac{n_2\eta_{12}}{n_2\eta_{2d} + n_1\eta_{21}} \tag{18}$$

and thus

$$\frac{\eta_{12}}{\eta_{2d}} = \frac{n_2 E_2}{n_2 E_1 - n_1 E_2} \tag{19}$$

Equations (14)-(19) are only valid for direct excitation of Subsystem 1, with Subsystem 2 being excited indirectly via the coupling joint. If the experiment is reversed and Subsystem 2 is directly excited with Subsystem 1 being excited indirectly via the coupling joint, then

$$\frac{\eta_{12}}{\eta_{1d}} = \frac{n_2 E_1}{n_1 E_2 - n_2 E_1} \tag{20}$$

Equations (19) and (20) allow one to set up experiments to measure the coupling loss factors between two subsystems provided that one has prior information about the modal densities and the internal loss factors of the individual subsystems. Firstly, Subsystem 1 is excited at a single point and the time- and space-averaged energies of vibration of the coupled subsystems are obtained with an accelerometer. The point excitation is repeated at several points, randomly chosen, to satisfy the assumption of statistical independence. Equation (19) is then used to obtain the coupling loss factor η_{12} and Equation (6) is subsequently used to obtain the coupling loss factor η_{21}. Alternatively, if Subsystem 2 was excited, Equation (20) could have been used. Information about the modal densities is obtained separately either experimentally or from theoretical relationships. Information about the internal loss factors is generally obtained from experiments. Very little theoretical information is available about internal loss factors; most available information is empirical and is based upon experimental data.

4. Applications of SEA

SEA was developed initially for aerospace applications and has been used very successfully in that area for calculating the vibratory response of complicated structures involving structure-to-structure vibration transmission, and also for structure/acoustic field interactions for estimating sound radiation and noise levels. SEA models involving both structural and airborne transmission have also been used in noise control on board ships. Among many applications where SEA are used, there are four major: (a) cars, (b) trains, (c) ships and offshore structures, (d) aeroplanes.

5. Conclusions

Statistical Energy Analysis is an energy dissipation based approach. It provides an alternative form of model that represents the average response behaviour of a population of systems and all results are expected values. The main idea of SEA approach is that one has to divide analyzed whole system structure into “subsystems”. All energy analysis is done between those subsystems. The vibrational state is expressed in terms of vibrational energies of individual components, where applied excitations are expressed in terms of input powers and the couplings between components are expressed in terms of energy flows. SEA is particularly appropriate in applications involving relatively large and lightweight structures, such as those designed for aerospace use. These statistical models are also helpful to mechanical designers who are charged with making environmental and vibratory response estimates at a stage in a project where structural detail is not yet known. SEA has some key features, which make this method very attractive as far as customer is concerned: (a) the power - energy relation is not so sensitive to small parameter change, (b) energy quantities can be averaged more easily, and (c) short modelling time. Moreover, SEA provides an approach to a number of vibration problems that cannot, from a practical viewpoint, be solved by classical methods.

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