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Modulation of a Periodic Object with Infinitely Thin Lines by a Human Eye in Presence of Stiles-Crawford Effect of the First Kind Using Coherent Light

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Abstract : The spatial frequency response of a human eye apodized with Stiles-Crawford effect of the first kind is studied under the condition of the duty cycle of a periodic object with triangular transmission profile tending to zero thereby consisting of only infinitely thin lines. First, the intensity distribution in the diffraction images of this special object is obtained by considering the human eye as being apodized with the Stiles-Crawford effect of the first kind. Next, the incident illumination is taken to be spatially coherent. And finally, the modulation is computed to ascertain the quality of the image quantitatively. We have shown that in agreement with the recent experimental finding the modulation of a periodic object with infinitely thin lines by a human eye in presence of Stiles-Crawford effect of the first kind is insensitive to coherent illumination or the SCE I apodization does not improve the quality of the image in the entire range of spatial frequency under spatially coherent illumination.

1. Introduction

In the Stiles-Crawford effect of the first kind (SCE I), a narrow beam entering at the edge of the pupil stimulates the retina less compared to a beam passing near the centre [1]. The same diminution of effective brightness in an image is observed in a pupil gradually becoming less transparent from the centre to its edge [2]. Thus, in spite of the SCE I being retinal in origin [3], it is modelled as a pupil apodization in computing diffraction images in the eye [4-10]. For diffraction image calculation though a sinusoidal grating producing sinusoidal variations of brightness in the image is best suited for a test target [11-14], the difficulty in obtaining such a grating of unit contrast over a wide frequency range [15-16] has led to the use of objects with square, rectangular and triangular waveforms as test targets [17-20].Similarly the choice of illumination was inclined more towards the use of the completely incoherent type, both non uniform [17-20] and uniform [21-23] for the richness it offered in terms of image quality compared to coherent illumination. No work, however, appears to have been done which simultaneously combine a) SCE I as pupil apodization, b) completely coherent illumination as incident light and c) a periodic object with infinitely thin lines as a test target. This paper reports the results obtained due to this treatment.

II. Theory

The double diffraction process of coherent imagery adopted here can be outlined as follows: An object with a complex amplitude distribution $A(x, y)$ is Fourier transformed to produce its corresponding spatial frequency spectrum $a(u,v)$. The apodizer, here SCE I modifies the object spectrum and the modified object spectrum $[a'(u,v)=a(u,v)\, f\,(u,v)]$ is inverse Fourier transformed to get the image complex amplitude distribution, $A'(x, y)$. Finally the squared modulus of this amplitude gives the intensity distribution in the image $B'(x, y)$ from which the modulation $(M = \frac{B'_{\max} - B'_{\min}}{B'_{\max} + B'_{\min}})$ is computed to ascertain quantitatively the quality of image.

The amplitude transmittance of a triangular wave grating, modelled as a transmitting structure in the object space with average amplitude *a*, modulation *b*, period *p* and duty cycle α can be expressed as follows: [18-20]

$$A(x, y)=(a-b+2\alpha b)+\frac{4b}{n^2\alpha}\sum_{n=1}^{\infty}\frac{\sin^2(n\alpha\pi)}{n^2}\cos(n\omega x) \tag{1}$$

where $\omega = \frac{2\pi}{p}$ .and *x*, *y* are the horizontal and vertical coordinates of space respectively. Along the other axis of the grating (parallel to the bars), the amplitude is fixed; thus, $A(x,y) = A(x)$ for all *y*. When $\alpha \to 0$, a periodic object with triangular wave profile reduces to an object with infinitely thin lines. In order that a finite quantity of light is transmitted, *b* should tend to infinity. Consideration of the normalized intensity in the line as unity, that is, $2\alpha b \to 1$, reduces (for $a = b$) the object function in Eq. 1 to

$$A(x)=1+2\sum_{n=1}^{\infty}\cos(n\omega x) \tag{2}$$

The Fourier transform of Eq. (2) leads to

$$a(u,v)=\iint_{-\infty}^{\infty} A(x,y)\, e^{-i(ux+vy)} dx\, dy$$

$$=\iint_{-\infty}^{\infty} \left[1+2\sum_{n=1}^{\infty}\cos(n\omega x)\right] e^{-i(ux+vy)} dx\, dy \qquad (3)$$

Thus Eq. (3) reduces to

$$=\iint_{-\infty}^{\infty} e^{-i(ux+vy)} dx\, dy + 2\sum_{n}\iint_{-\infty}^{\infty} \cos(n\omega x)\, e^{-i(ux+vy)} dx\, dy$$

$$=\delta(u,v)+\sum_{n}\iint_{-\infty}^{\infty}\left[e^{-i[(u-n\omega)x+vy]}+e^{-i[(u+n\omega)x+vy]}\right] dx\, dy$$

$$=\delta(u,v)+\sum_{n}\,[\delta(u-n\omega,v)+\delta(u+n\omega,v)]$$

$$a(u,v)=\delta(u,v)+\sum_{n}^{\infty}\,[\delta(u-n\omega,v)+\delta(u+n\omega,v)] \qquad (4)$$

Given that the input to the pupil is $a(u,v)$, the Fourier transform of the pupil exit function will be $f(u,v)\,a(u,v)=a'(u,v)$ for f being the amplitude transfer function which approximate as follows [24]: The SCE I modified visibility is empirically represented by a Gaussian function as $\eta=\exp(-\rho_e r^2)$ where $\rho_e=\frac{1}{\sigma^2}=0.105$ /mm^2 and $\sigma=3.086$ (ρ_e is the directionality coefficient that measures the width of the pupil apodization) , r is distance in the entrance pupil from the origin of the function and η is the visibility. So,

$$f(u,v)=e^{\frac{-[(u]^2+v^2)}{\sigma^2}}=e^{-0.105(u^2+v^2)}\; n'\le\frac{1}{\omega}$$ for coherent illumination [25]. Hence,

$$a(u,v)=e^{\frac{-[(u]^2+v^2)}{\sigma^2}}\left[\delta(u,v)+\sum_{n}^{\infty}\,[\delta(u-n\omega,v)+\delta(u+n\omega,v)]\right] \qquad (5)$$

The image amplitude distribution at the exit pupil is obtained by the inverse Fourier transform of the spectrum $a'(u,v)$. Thus

$$A'(x,y)=\iint_{-\infty}^{\infty} a'(u,v)\, e^{i(ux+vy)} du\, dv$$

$$A'(x,y) = \int\int_{-\infty}^{\infty} e^{\frac{-[(u]^2+v^2}{\sigma^2}} \left[\delta(u,v) + \sum_{n}^{\infty} \left[\delta(u-n\omega,v) + \delta(u+n\omega,v\right] \right] du\, dv \qquad (6)$$

Utilizing again the properties of the Dirac delta function the above equation can be simplified to

$$A'(x,y) = f(0,0) + \sum_n f(n\omega,0)e^{in\omega x} + \sum_n f(-n\omega,0)e^{-in\omega x}$$

$f(0,0)$ can be normalized to unity and for rotationally symmetric system we can write

$$f(n\omega,0) = f(-n\omega,0)\,. \quad \text{As} \quad f(n\omega,0) = e^{\frac{-n^2\omega^2}{\sigma^2}}$$

Hence

$$A'(x,y) = 1 + \sum_n f(n\omega,0)e^{in\omega x} + \sum_n f(n\omega,0)e^{-in\omega x} = 1 + 2\sum_n f(n\omega,0)\cos(n\omega x)$$

$$A'(x,y) = 1 + 2\sum_{n=1}^{n'} f(n\omega,0)\cos(n\omega x) \qquad (7)$$

As $f(n\omega,0) = 0$ for $n\omega \geq 1$ the upper limit n' of n is such that $n\omega \leq 1$. Finally, the image intensity distribution will be given by the squared modulus of above, that is,

$$B'(x,y) = [A'(x,y)]^2$$

$$B'(x,y) = \left[1 + 2\sum_{n=1}^{n'} e^{\frac{-n^2\omega^2}{\sigma^2}} \cos(n\omega x) \right]^2 \qquad (8)$$

The modulation in the image (for unit contrast in the object) can be defined as

$$M = \frac{B'_{\max} - B'_{\min}}{B'_{\max} + B'_{\min}} \qquad (9)$$

ωx is varied from 0 to π to compute the maximum and minimum intensity.

III. Results and Discussion

The intensity distribution in the images has been calculated by use of Eq. (8).The method of computation was straightforward. Eq. (8) was programmed to evaluate intensity for a large number of values of $x(0 \le x \le \frac{p}{2})$ and the final results were then plotted. The normalized spatial frequency (ω) is taken in steps of 0.1. The dependence of intensity on reduced distance $(\frac{2x}{p})$ is thus illustrated in Figs.1 through 5 and in Fig. 6 for both low and high normalized spatial frequency. These indicate how the image behaves for various values of normalized spatial frequency (ω) for a diffraction-limited and aberration-free human eye apodized with the Stiles-Crawford effect of the first kind. One can notice that with the increase of ω the image intensity distribution pattern rapidly rounds to a smooth pattern from an abrupt one as is revealed from the sequence of histograms plotted (Fig.1-5) or to a cosine term (Fig. 6).

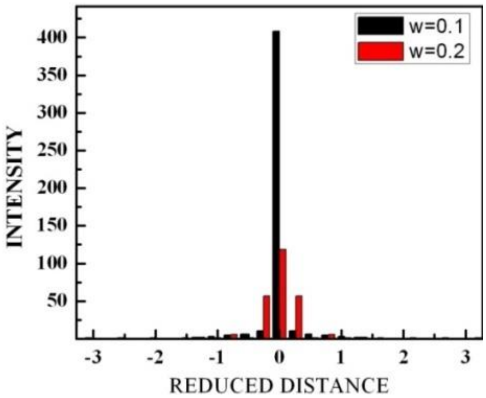

Fig. 1

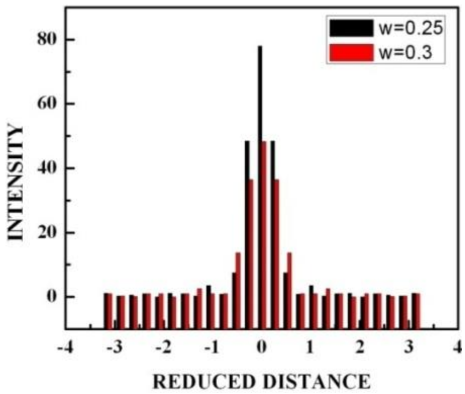

Fig. 2

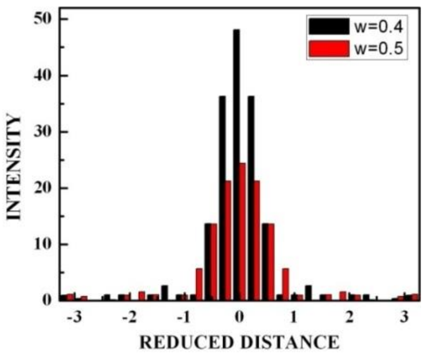

Fig. 3

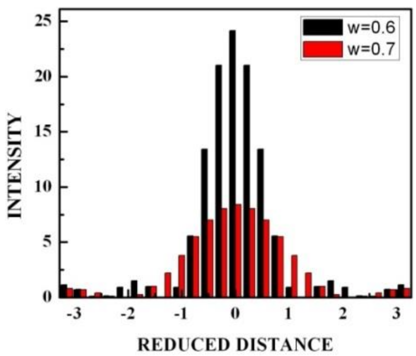

Fig. 4

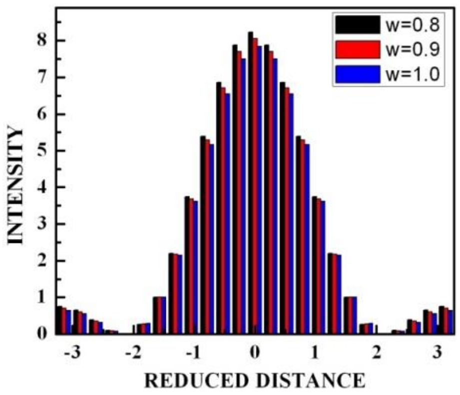

Fig. 5

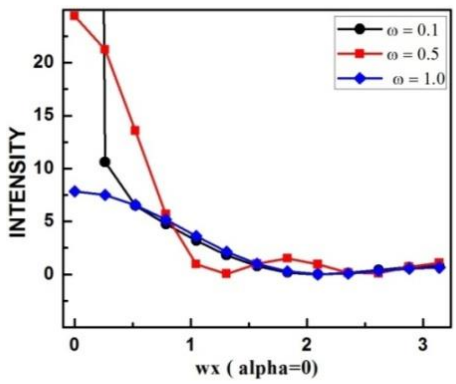

Fig.6: intensity vrs reduced distance

In order to ascertain the quality of the image the frequency response is evaluated in the form of modulation as defined in Eq. (9). Again this formula is programmed to obtain the values tabulated below.

Table-1

ω	$B'_{\max}$	$B'_{\min}$	$B'_{\max} - B'_{\min}$	$B'_{\max} + B'_{\min}$	$M = \frac{B'_{\max} - B'_{\min}}{B'_{\max} + B'_{\min}}$
0.1	408.7938	4.29828E-05	408.7938	408.7939	1
0.2	118.3588	0.014583753	118.3442	118.3734	0.999754
0.25	77.97781	0.007624097	77.97019	77.98544	0.999804
0.3	48.35803	0.026683411	48.33134	48.38471	0.998897
0.4	48.15645	0.027924341	48.12852	48.18437	0.998841
0.5	24.40161	0.048648527	24.35296	24.45025	0.996021
0.6	24.18107	0.051236437	24.12983	24.23231	0.995771
0.7	8.408249	0.002515121	8.405734	8.410764	0.999402
0.8	8.236963	0.004224291	8.232738	8.241187	0.998975
0.9	8.048149	0.006648288	8.0415	8.054797	0.998349
1	7.843589	0.009936019	7.833653	7.853525	0.99747

The modulation is plotted against the normalized spatial frequency to obtain Fig. 7. From the graph it is evident that the modulation decreases with the increase of the spatial frequency, then attains a minimum contrast for a particular spatial frequency of ω=0.6 and finally falls sharply with ω. But the contrast changes only by 0.4 % throughout the entire range of spatial frequency indicating almost ineffectiveness of modelling SCE-I as a pupil apodization for a human eye under coherent illumination.

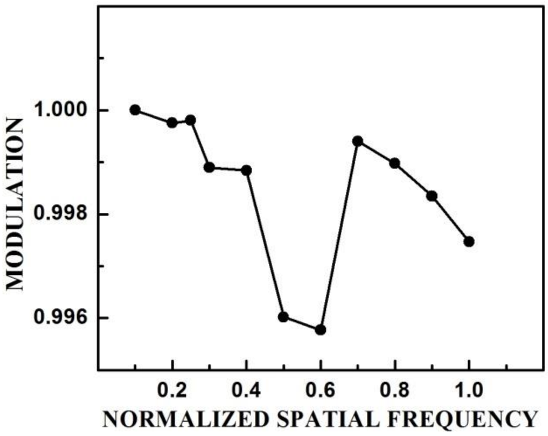

Fig.7: Variation of modulation with ω

iv. Conclusion

The result thus arrived at taking into account a periodic object with infinite lines points to an important conclusion that the modulation is insensitive to spatial frequency under coherent illumination. It also indirectly strengthens the experimental finding [26] of the absence of the traditional SCE-I directionality by bringing into the forefront the conclusion that Stiles-Crawford apodization in coherent illumination is not an influencing factor for a human eye [10,27].

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