PROGRAMMING OF THE METHOD FOR THE SOLUTION OF LINEAR GOAL PROGRAMMING PROBLEMS

1.1 Introduction

Now-a-days, the managers in the Corporations and companies face troubles in practical situations involving multiple, competitive and conflicting goals. Generally any programming problem is concerned with the efficient use of allocation of limited resources to meet the desired objectives. Such types of problems have large number of solutions satisfying the basic conditions of the problem. Out of these solutions, the best solution to the problem is that which satisfies the basic conditions and the given objective function. This particular solution is called the optimum solution.

Decision making is the process of selecting a possible course of actions from all the available alternatives. The decision maker wants to attain more than one objective or goal while satisfying the constraints.

Although theoretical results had been worked out for the solution of many types of problems earlier, but they could not be applied to "real" problems because of the amount of calculations involved. The advent of computers has made its impact and has rendered the possibilities of solution of real problems. The linear goal programming is gaining its popularity because it can be thought of as an extension of linear programming. Also it provides the facility to the decision maker to include the conflicting objectives while still obtaining a solution that is optimal with respect to the decision maker's specification of goal priorities. In addition, the single objective function problem can be solved by this method. The details regarding the goal programming are discussed in Huang and Masud (1979), Ignizio (1976), Lee (1972), Markland and Sweigard (1987) etc.

The modified simplex method is used for solving the linear goal programming problems.

1.2 Formulation of Linear Goal Programming Model

The key to goal programming problem formulation is that each goal or objective is written in the form of constraints. It may be under-achieved or fully achieved or over-achieved. Let us define,

d_i^+ = over-achievement of the ith goal

T_i^- = under-achievement of the ith goal. (1.2.1)

Since we can not have both under-achievement and over-achievement of a goal simultaneously, so either one or both of these deviational variables for ith goal will be equal to zero i.e. $d_i^+ \cdot d_i^- = 0$.

Therefore, the target level of ith goal b_i is fully achieved iff

$$\sum_{j=1}^{n} a_{ij}\, x_j = b_i\,. \tag{1.2.2}$$

Hence each ith goal can be represented as

$$\sum_{j=1}^{n} a_{ij}\, x_j + d_i^- - d_i^+ = b_i \quad \text{for } i = 1,2,\ldots, m \tag{1.2.3}$$

Our model is able to incorporate goals with both ranking and weighting as appropriate i.e. the decision maker will inherently favour the achievement of one goal over another. The goals are given an ordinal ranking which are called pre-emptive priority factors. These priority factors have the relationship

$$P_1 > P_2 > P_3 > \ldots > P_{j+1} \tag{1.2.4}$$

where > represents that P_1 goal is more important than P_2 and P_2 is more important than P_3 and so on. So P_2 goal will never be attempted until the P_1 goal is achieved to the greatest extent possible. This process continues till the last goal is achieved. In formulating a goal programming model, these pre-emptive priority factors are incorporated in to the objective function as weights for the deviational variables.

Now the linear goal programming model may be formulated as

Minimize

$$Z = \sum_{i=1}^{k} \sum_{j=1}^{m} P_i \left(W_{ji}^+\, d_j^+ + W_{ji}^-\, d_j^- \right) \tag{1.2.5}$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j - d_i^+ + d_i^- = b_i \quad \text{for } i = 1,2, \ldots, m \tag{1.2.6}$$

where

$$d_i^+ \cdot d_i^- = 0 \tag{1.2.7}$$

$W_{ji}^+ =$ relative weight of the d_j^+ in the ith rank.

$W_{ji}^- =$ relative weight of the d_j^- in the ith rank.

k = number of goals

m = number of constraints including goal constraints and absolute constraints

n = number of variables.

In this chapter we consider the paper by Basu and Pal (1985) in which the authors have considered the Goal Programming model for long range resource planning with particular emphasis on personnel management problems. Our aim is to frame the computer program in FORTRAN for implementing the model considered by Basu and Pal (1985) and make the testing of the program. Before this we shortly discuss the algorithm.

1.3 Algorithm for the Solution of Linear Goal Programming Problem (GPP)

The simplex algorithm can be used for optimization of linear goal programming model (1.2.5) – (1.2.6) using the following steps.

(A) The Initial Matrix

Linear Goal Programming requires the minimization of a linear weighted ranking function subject to certain linear constraints. Because these constraints are expressed in goal form by the use of deviational variables, they are equalities of the form specified by general linear programming model. Thus the deviational variables in linear goal programming model play the same role as that of slack on artificial variables instead of slack or artificial variables.

(B) Optimality Test

After an initial feasible solution has been obtained, it must be tested for optimality. This test is accomplished with the following steps.

i) Calculate Z_j for matrix B by $\sum_{i=1}^{m} c_i\, b_i$.

ii) Calculate $Z_j - c_j$ by $\sum_{i=1}^{m} c_i\, a_{ij} - c_j$ for j = 1,2,3,…, n,

where c_j gives the weighted ranking coefficients applicable to the initial feasible solution.

iii) All the $Z_j - c_j$ coefficients in the row should be examined to see if there are negative entries at higher priority levels in the same column. Such negative entries signify that an optimal solution has not been obtained.

iv) If the attainment level for each goal in b_i column is zero, the solution is certainly optimal. But if a positive value exists for one or more unattained priority levels, a better solution may be possible.

(C) Interactions towards an Optimal Solution

If the optimal test indicates that an optimal solution has not been found, the following interactive procedure may be employed.

i) Find the highest priority level that has not been attained by examining the $Z_j - c_j$ values of b_i columns. Then find the largest positive $Z_j - c_j$ without a negative value at a higher priority. Let this column be *kk*. The variable at the head of this column will be incoming variable for selection of the pivot element.

ii) Calculate the ratio $\theta_i = \dfrac{b_i}{a_{ikk}}$ for i = 1,2,3,…, m.

iii) Calculate the minimum of only positive θ_i. Let the minimum θ_i has row L. This is the pivot row.

iv) Nor replace old a_{ij} by new a'_{ij} by

$$a'_{ij} = \frac{a_{Lj}}{a_{Lkk}} \quad \text{for } i = L$$

and $$a'_{ij} = a_{ij} - \frac{a_{ikk}}{a'_{Lj}} \text{ for } i \neq L. \tag{1.3.1}$$

for $i = 1,2,\dots\ m$ and $j = 1,2,\dots, n$.

v) As in case of (iv) now replace old b_i by b'_i as

$$b'_i = b_i - \frac{b_i}{a_{Lkk}} \text{ for } i \neq L$$

$$b'_i = \frac{b_i}{a_{Lkk}} \text{ for } i = L \tag{1.3.2}$$

where $i =$ 1,2,…, m .

1.4 Special Problems in Linear Goal Programming

Since we are using the simplex procedure for solving the linear goal programming problem, so a number of special problems may arise as in case of linear programming. Those are discussed below.

(A) Alternative Optimal Solutions

This can be detected from the presence of zeros in an entire column of $Z_j - c_j$ for a non-basic variable with the existence of at least one positive a_{ij} in the corresponding column.

(B) Unbounded Solutions

Since every goal is constrained and the goals are set at a level that is not easily reached, so possibility for this situation is very less. If it occurs in any case, then the decision maker should check whether he may have omitted an important constraint.

(C) Infeasible Solutions

Here infeasibility may not occur because of the presence of deviation variables in the constraints. This will be detected from a positive $Z_j - c_j$ value for some non-basic variable associated with the highest priority level. In this case, the decision maker has to modify the absolute objectives or the problem environment.

(D) Tie for Entering Basic Variable

When we go for selection of max $Z_j - c_j$ for all positive $Z_j - c_j$, there is every possibility that we may get one or more *j*. In this problem, we can consider any *j*.

(E) Tie for Leaving Basic Variable

The variable to be removed from the basis is determined by taking smallest positive θ_i. If two or more rows have the same ratio then the ratio having the highest priority level associated, will be preferred for breaking the tie.

1.5 Program Discussion

The program for getting the solution of a linear goal programming problem is framed in FORTRAN language. The main program is divided into various parts and different subroutines have been constructed for different tasks. Depending on the efficiency of the computer available, the program framed can accommodate fifty goals with fifty constraints involving fifty variables. The constraints include both goal constraints and absolute constraints. The variables include deviational variables and absolute variables. However, this number can be expanded by slightly modifying the program (i.e. the DIMENSION specifications in all the subroutines and main program) if the machine available has larger capacity. The function of the subroutines in the program is listed as follows :

DENTRY : Used for entering of data through the console and preparation of the Initial Matrix as described at 1.3 (A).

BASIS : Used for generation of the basis to be used in every phase of iteration.

GENTRK : Used for searching of the pivot column *kk*.

TRACEL : Used for tracing of pivot row *L*.

CHANGX : Changes the contents of a_{ij} and b_i according to equn. (3.3.1) and (1.3.2) respectively.

FINAL : Prints the results of the solved problem.

The structure of the subroutines are general and they can be used for similar problems on resource planning etc. It is pertinent to mention here that since the role of artificial slack variables is not there, the subroutine for this purpose is avoided. The program is designed according to the requirement as discussed in § 1.3.

☞ [PROGRAM]

1.6 Numerical Testing of the Program

For testing of the program we use the data given by Basu and Pal (1985). These data concern the resource planning of personnel management in University.

The computed results are same as those obtained by Basu and Pal (1985). The problem is stated shortly as follows.

A numerical example is presented to demonstrate the application of the proposed GP model. This example is based on the actual operational data at a certain university. To simplify the example, only two units are taken in to consideration. The data of the staff at a particular period of one year are summarized in Table 1.1.

Table – 1.1

Unit	Number of Staff	
	Unit – I	Unit – II
TS		
Professor (Rank 3)	2	1
Reader (Rank 2)	4	1
Lecturer (Rank 1)	12	4
Total	18	6
NTS	6	6
RF	6	0

The Table 1.2 shows the annual salaries, increment and promotion rates. These are constant for all units.

Table – 1.2

	Annual salary (Rs. 1000's) (Average)	Increment Rate	Promotion Rate
Professor	30	0.12	-
Reader	25	0.12	0.34
Lecturer	20	0.12	0.34
NTS	10	0.25	-
RF	7.2	-	-

The Table 1.3 indicates the desired goal level of TS, NTS-TS ratio and RF-TS ratio, based on which the decision variables will try to achieve in the next planning period (say period 2).

Table – 1.3

Unit	Desired Goal level (TS)	NTS-TS Ratio	RF-TS Ratio
Unit-I	20	1:3	1:3
Unit-II	10	2:3	1:3

The total budget for the period 2 is Rs.922,000.00

Now the intention of the institution is to offer the promotion facility to the existing TS and to employ the new TS at the lowest rank. Hence, the new TS at rank 1 will be a variable in the model at the period 2.

Based on the approved data, the goal constraints at the period 2 can be expressed as

$$x_{112} + d_1^- - d_1^+ = 2$$

$$x_{212} + d_2^- - d_2^+ = 4$$

$$3z_{12} - x_{112} + d_3^- - d_3^+ = 0 \tag{1.6.1}$$

$$2x_{212} - 3z_{22} + d_4^- - d_4^+ = 6$$

$$3R_{12} - x_{112} + d_5^- - d_5^+ = 0$$

$$3R_{22} - x_{212} + d_6^- - d_6^+ = 6$$

$$5x_{112} + 5x_{212} + 2.5z_{12} + 2.5z_{22} + 1.8R_{12} + 1.8R_{22} + d_7^- - d_7^+ = 44.5$$

The objective function is

$$\text{Min } P_1\left(d_1^- + d_2^-\right)$$

$$\text{Min } P_2\left(d_3^- + d_4^-\right) \tag{1.6.2}$$

$$\text{Min } P_3\left(d_5^- + d_6^-\right)$$

$$\text{Min } P_4\left(d_7^+\right)$$

Let

X_{ijt}	=	number of TS in academic unit *i*, rank *j*, at the period *t*,
R_{it}	=	number of RF in unit *i*, at the period *t*,
Z_{it}	=	number of NTS in unit I, at the beginning of the period *t*+1,
$x_{ij,t+1}$	=	number of new TS employed in unit *i*, rank *j*, at the beginning of the period *t*+1,
r_{it}	=	ratio of Z_{it} to X_{ijt} ,
q_{it}	=	ratio of R_{it} to X_{ijt} ,
G_{it}	=	goal target for X_{ijt} ,
S_{ijt}	=	salary for X_{ijt} ,
s_{it}	=	salary for Z_{it} ,

c_{it} = cost for R_{it} ,

B_t = total projected budget at the period t,

$\alpha_{ij-1,t}$ = proportion of X_{ijt} promoted out of rank j-1 (to rank j),

from period t to t+1

β_{ijt} = promotion of X_{ijt} who remain in the rank j , from period t to $t + 1$.